CHAPTER 03: MORTGAGE LOAN FOUNDATIONS: THE TIME VALUE OF MONEY

Future Value

- Compound Interest
  - Earning Interest on Interest
- Basic Components
  - *PV* = Initial deposit
  - *i* = Interest rate (some authors use “r”)
  - *n* = Number of years
  - *FV* = Value at a specified future period

Future Value

General equation:

$$FV_n = PV(1+i)^n$$

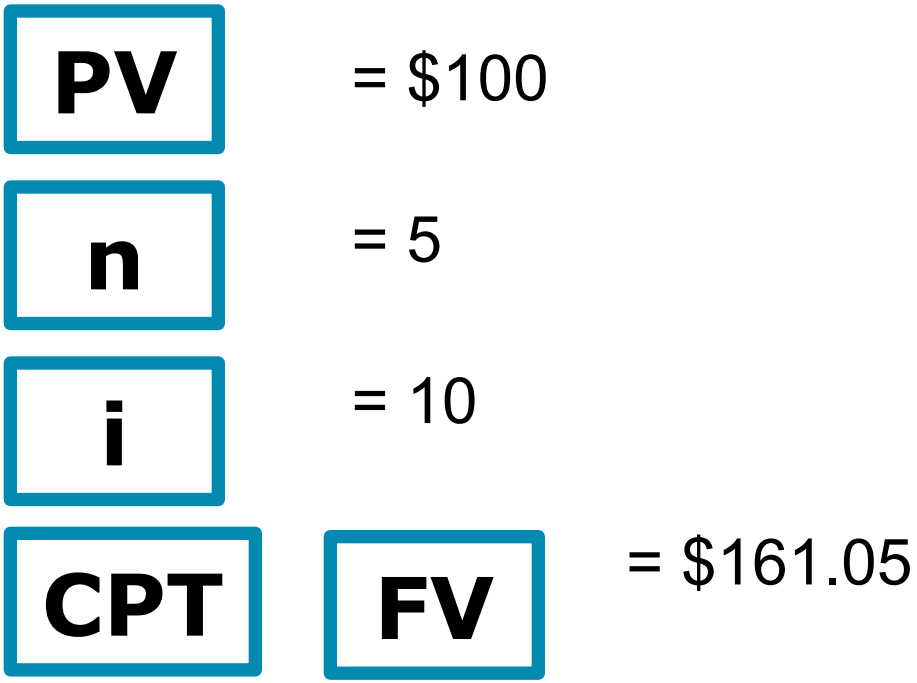
Future Value

- Example 3-1:
  - What is the value at the end of year 5 of $100 deposited today if the interest rate is 10% compounded annually?

$$FV_5 = \$100(1.10)^5$$
$$= \$100(1.61051)$$
$$= \$161.05$$

Future Value

- Example 3-1 Using a Financial Calculator:



Future Value

- Semi-Annual Compounding
  - In Example 3-1, what if interest were paid semi-annually instead of annually?
    - There would be two compounding periods in each year.
    - There would be a periodic rate to match the multiple compounding periods.
    - The time period would be doubled.
    - ***Most importantly, the future value would be higher. Additional compounding periods will effect the final result.***

Future Value

Our general equation becomes:

$$FV_n = PV\left[1 + \frac{i}{m}\right]^{n \cdot m}$$

where m = number of compounding intervals in a year

Future Value

- $\frac{i}{m}$ is also called the period rate
- For Example 1:

$$FV_5 = 100\left[1+\frac{.10}{2}\right]^{5\cdot 2}$$

$$= 100(1.62889)$$

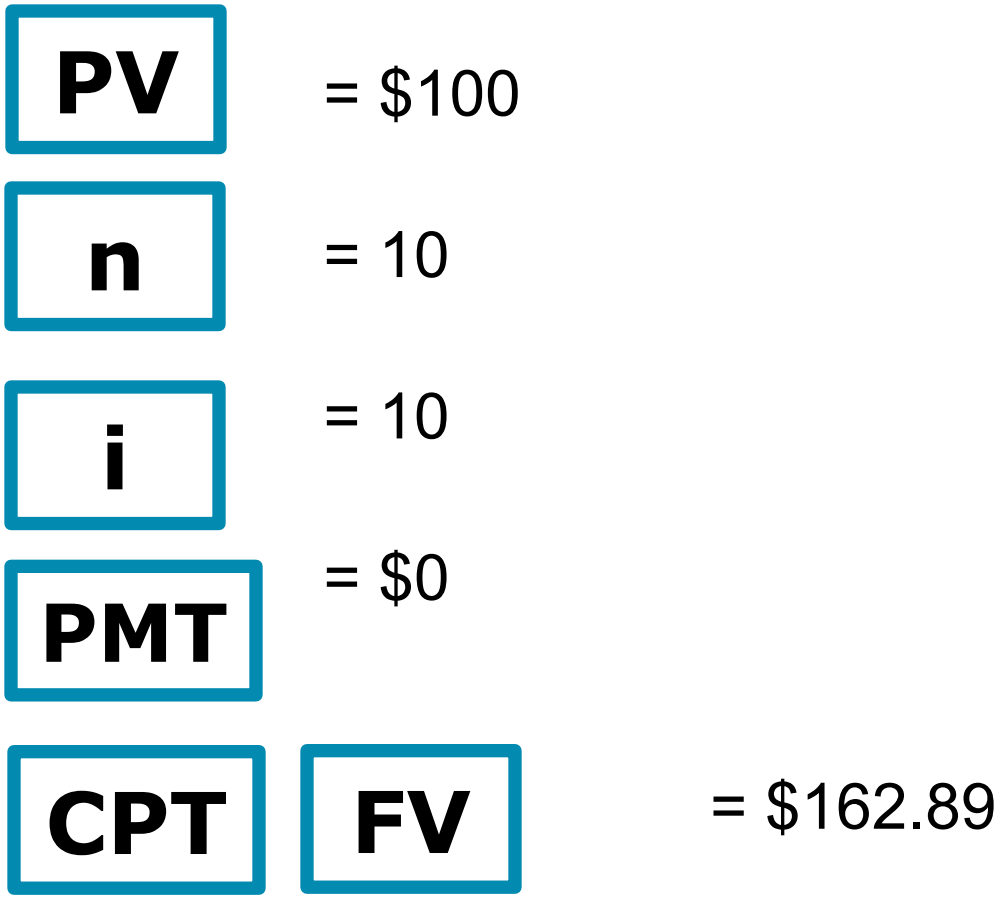
$$= \$162.89$$

Future Value

- Two alternatives for multiple compounding periods and most financial calculators
  - You can change P/Y to the number of compounding periods
    - Example: Change P/Y to 2 for semiannual compounding
  - You can enter a periodic rate
    - Example: Enter i/2 as the interest rate for semiannual compounding

Future Value

- If you change P/Y to 2, then



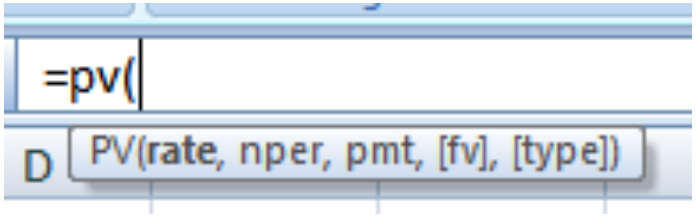
Future Value

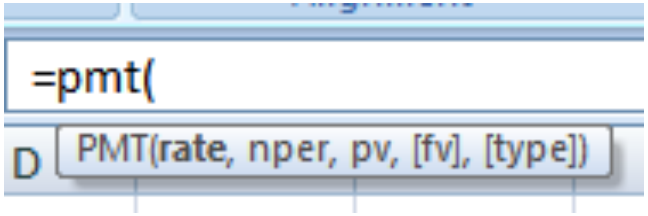
- Notice the difference in Future Value when multiple compounding periods are used:
  - $162.89 (semi-annual) vs. $161.05 (annual)
- This shows the effect of earning interest on interest. The more compounding periods there are per year, the higher the future value will be.

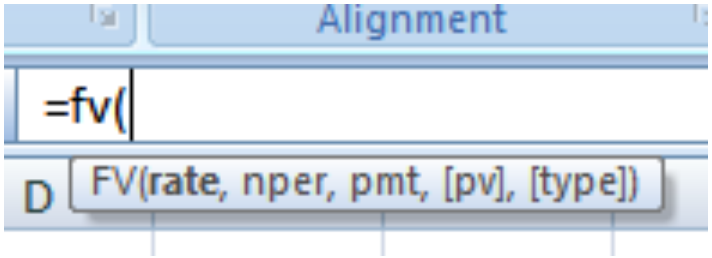
Financial Functions: Concept Box 3.1

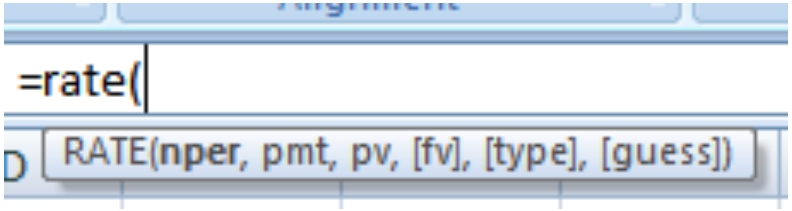
- *PMT* (*n, i, PV, FV*)
- *PV* (*n, i, PMT, FV*)
- *FV* (*n, i, PV, PMT*)
- *i* (*n, PV, PMT, FV*)
- *n* (*i, PV, PMT, FV*)

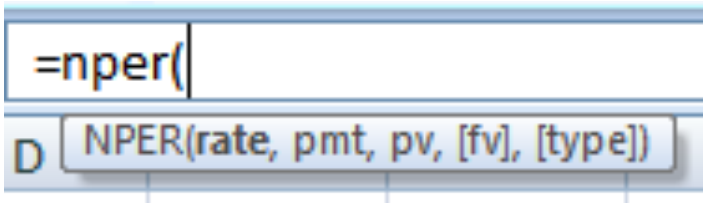
Financial Functions: Concept Box 3.1











For complex analysis, Excel is far superior to a financial calculator.

Present Value

- Discounting: Converting Future Cash Flows to the Present
- General equation

$$PV = FV_n \frac{1}{(1+i)^n}$$

Present Value

- Example 3-2:
  - What is the value today of $2,000 you will receive in year 3 if the interest rate is 8% compounded annually?

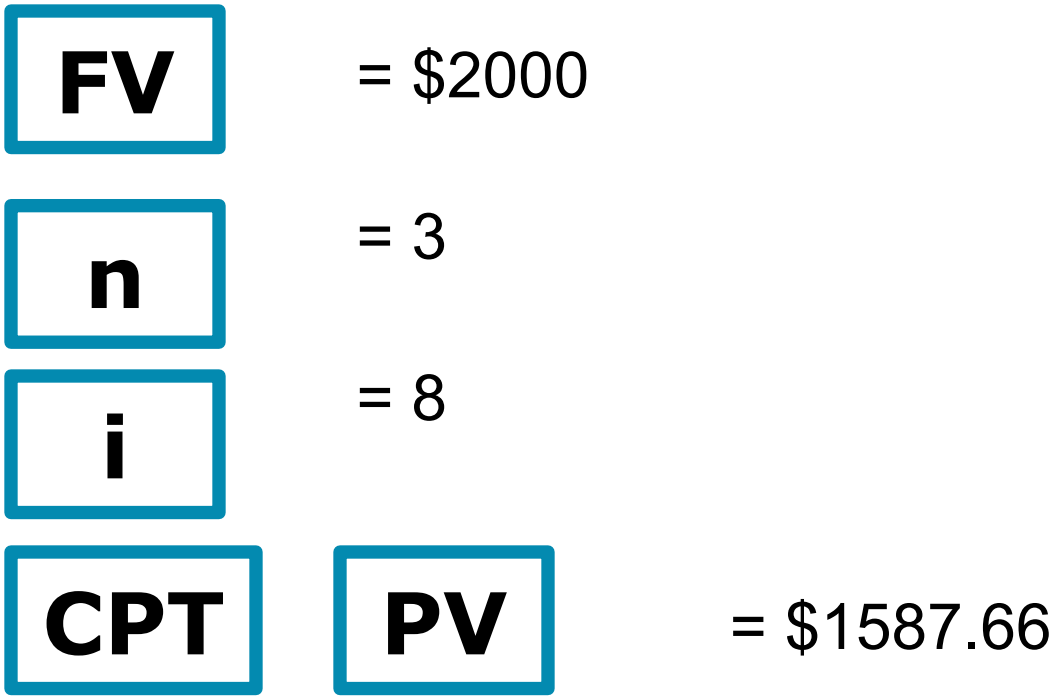
$$PV = 2000\left[\frac{1}{(1.08)^3}\right]$$

$$= 2000(.79383)$$

$$= \$1587.66$$

Present Value

- Example 3-2 Using a Financial Calculator:



Present Value

- Example 3-2 with 8% Compounded Monthly
- Mathematically:

$$PV = FV_n \left[\frac{1}{(1 + \frac{i}{m})^{n \cdot m}} \right]$$

Present Value

$$PV = 2000\left[\frac{1}{\left(1+\frac{.08}{12}\right)^{12\cdot 3}}\right]$$

$$= 2000(.78725)$$

$$= \$1574.51$$

Present Value

- If P/Y is changed to 12

FV = $2000

n = 36

i = 8

PMT = $0

CPT **PV** = $1574.51

Annuity

- Level Cash Flow Stream
- Terminates at some date in the future
- Ordinary Annuity
  - Cash flows begin one period from today
- Annuity Due
  - Cash flows begin immediately

Annuity: Future Value

- General Equation:

$$FV = P(1+i)^{n-1} + P(i+i)^{n-2} + \ldots + P$$

$$FV = P \cdot \sum_{t=1}^{n-1} (1+i)^{t} + P$$

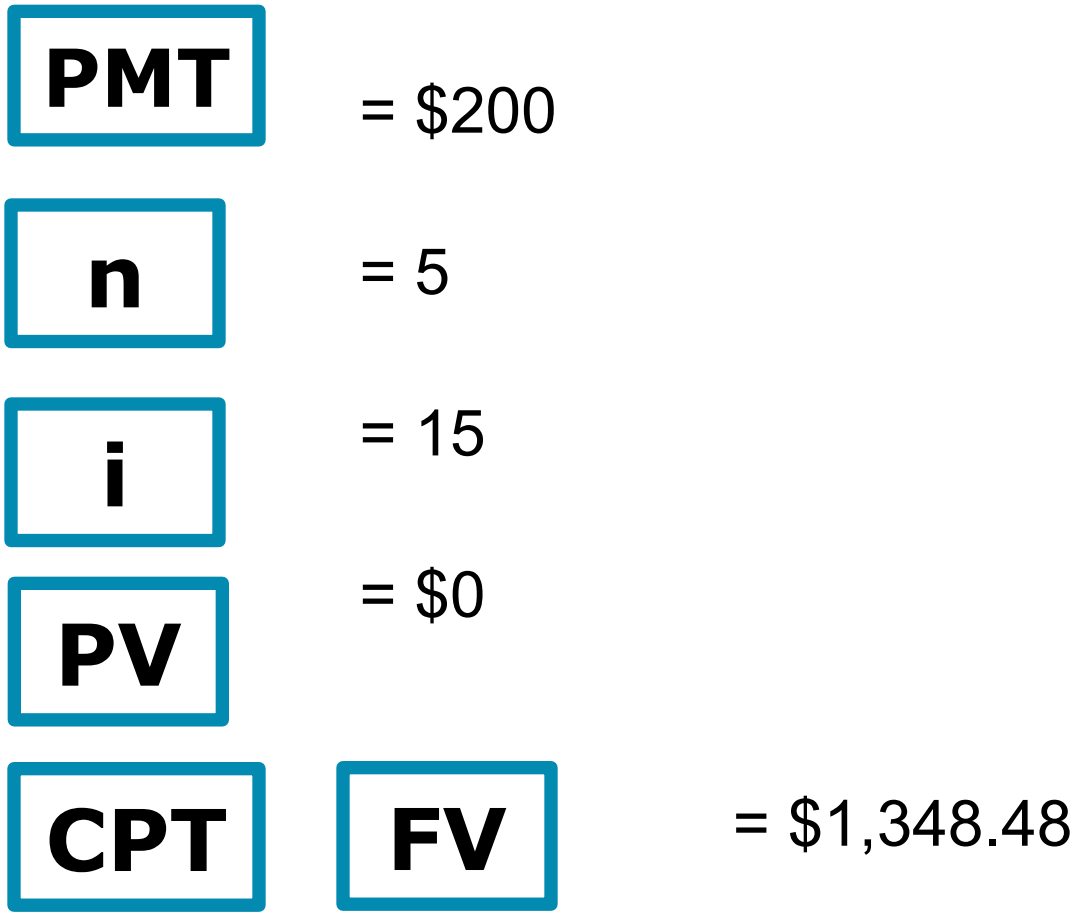
Annuity: Future Value

- Example 3-3:
  - What is the future value of a 5-year ordinary annuity with annual payments of $200, evaluated at a 15% interest rate?

$$FVA = 200 \cdot \frac{(1+.15)^5 - 1}{.15}$$
$$= 200(6.74238)$$
$$= \$1,348.48$$

Annuity: Future Value

- Using the Financial Calculator:



Annuity: Future Value

- For Example 3-3, if payments were to be received monthly
- Mathematically:

$$FV = P \cdot \left[1 + \frac{i}{12}\right]^{n \cdot m - 1} + P \cdot \left[1 + \frac{i}{12}\right]^{n \cdot m - 2} + \ldots + P$$

$$FV = P \cdot \sum_{t=1}^{n \cdot m - 1} \left[1 + \frac{i}{12}\right]^{t} + P$$

Annuity: Future Value

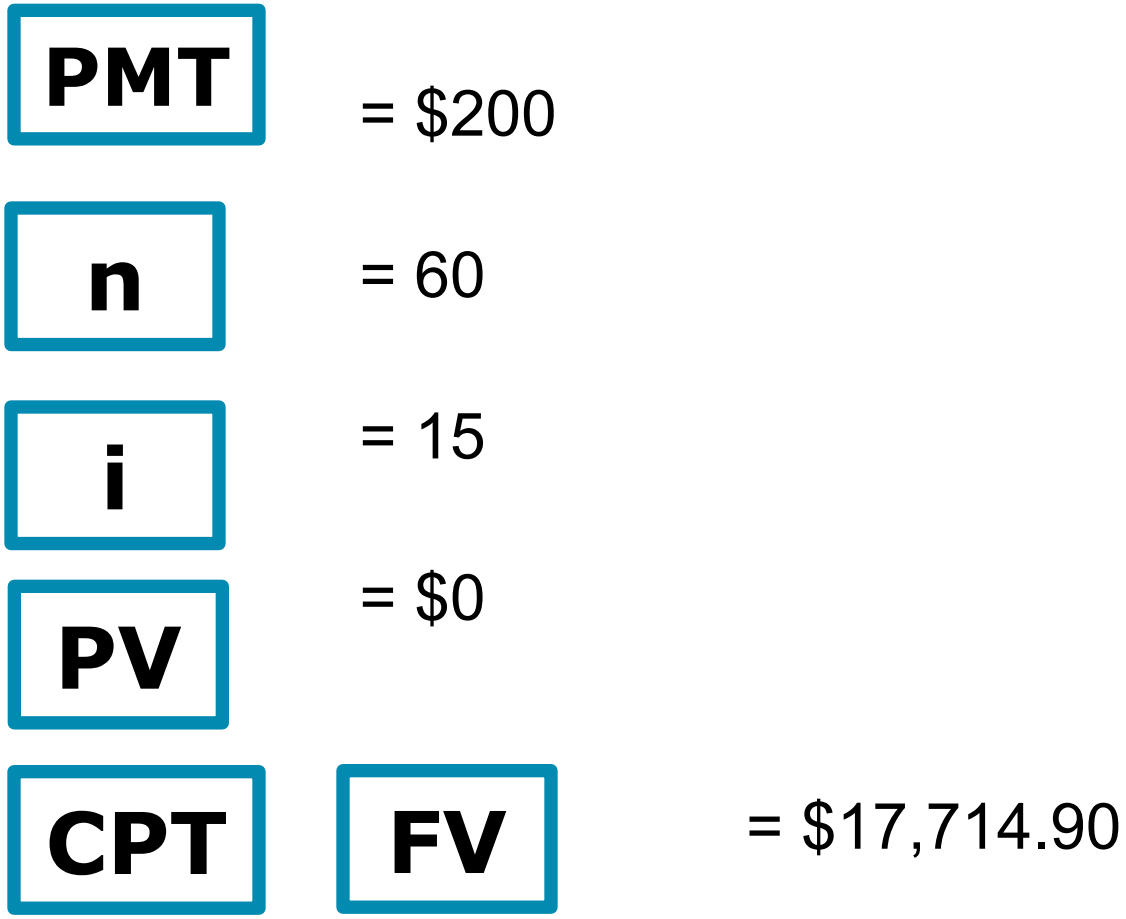
$$FV = 200 \cdot \left[\frac{\left(1 + \frac{.15}{12}\right)^{5 \cdot 12} - 1}{\frac{.15}{12}} \right]$$

$$= 200(88.5745)$$

$$= \$17{,}714.90$$

Annuity: Future Value

- Using the Financial Calculator, if P/Y = 12



Annuity: Present Value

- General Equation:

$$PV = PMT \cdot \frac{1}{(1+i)^1} + PMT \cdot \frac{1}{(1+i)^2} + \ldots + PMT \cdot \frac{1}{(1+i)^n}$$

$$PV = PMT \cdot \sum_{t=1}^{n} \frac{1}{(1+i)^t}$$

Annuity: Present Value

- Example 3-4:
  - If you had the opportunity to purchase a $500 per year, ten-year annuity, what is the most you would pay for it? The interest rate is 8%.

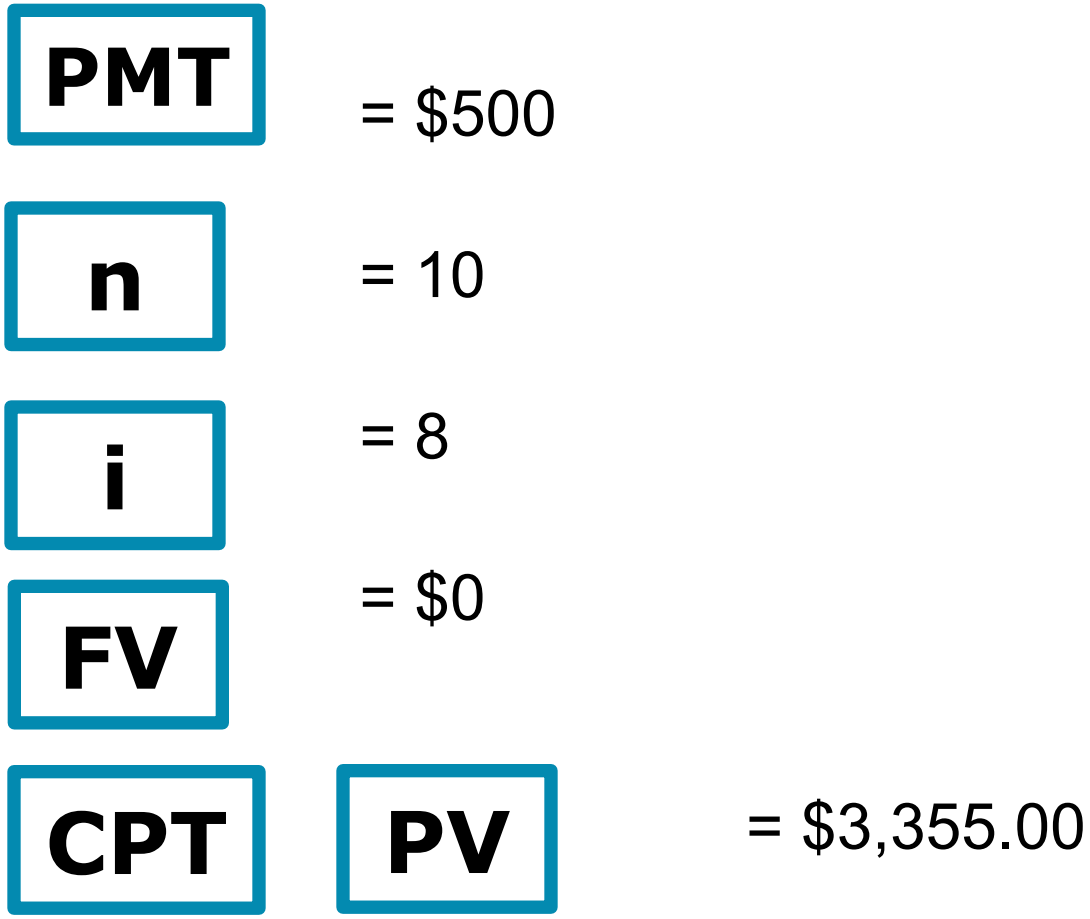
$$PVA = 500 \cdot \frac{\left(1 - \frac{1}{1.08^{10}}\right)}{.08}$$

$$= 500(6.7100)$$

$$= \$3355.00$$

Annuity: Present Value

- Using the Financial Calculator:



Annuity: Present Value

- For Example 3-4, if Payments Were to Be Received Monthly
- Mathematically:

$$PV = P\left[\frac{1}{1+\frac{i}{12}}\right]^{1} + P\left[\frac{1}{1+\frac{i}{12}}\right]^{2} + \ldots + P\left[\frac{1}{1+\frac{i}{12}}\right]^{12\cdot n}$$

Annuity: Present Value

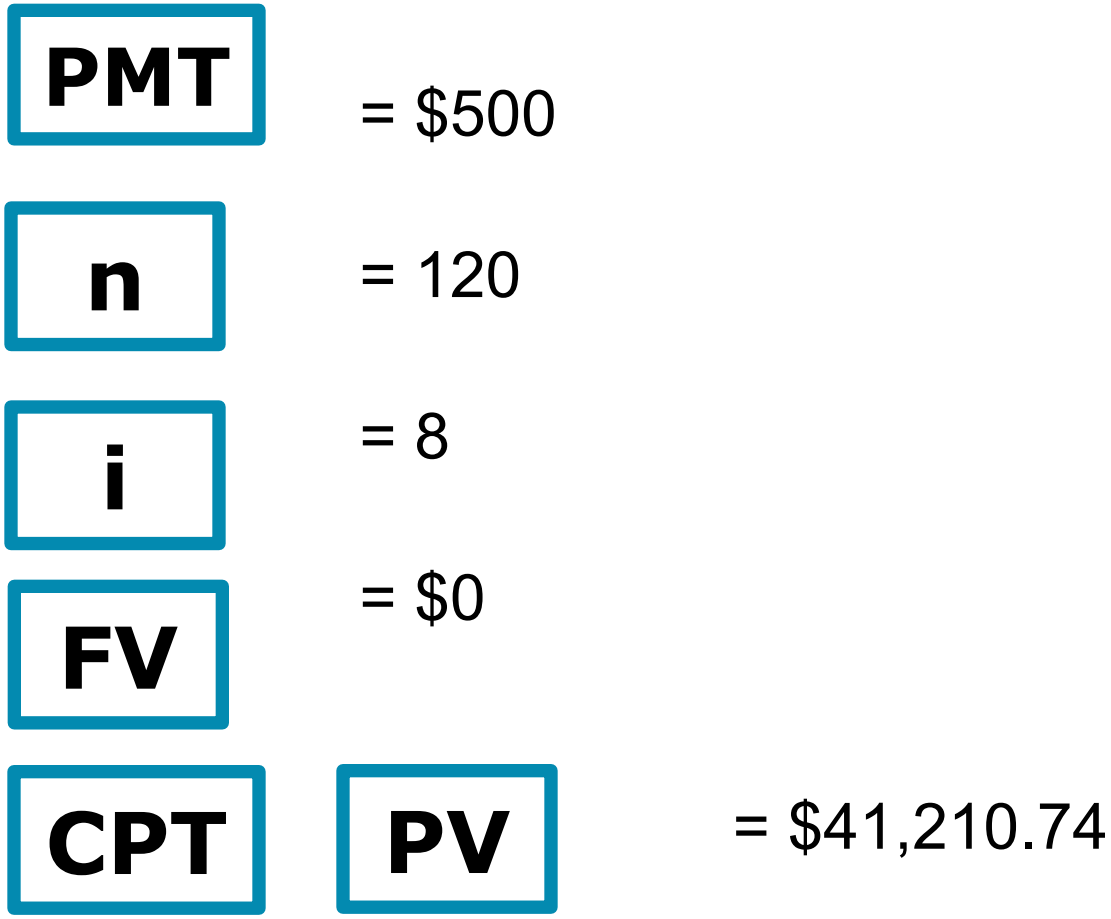
$$PVA = \$500 \cdot \left[\frac{1 - \frac{1}{(1 + \frac{.08}{12})^{120}}}{\frac{.08}{12}} \right]$$

$$= \$500(82.4215)$$

$$= \$41,210.74$$

Annuity: Present Value

- Using the Financial Calculator, if P/Y = 12



Time Value of Money—Extensions

- Given the basic equations that we have discussed, we can solve for any missing single variable.
- Some common applications
  - Solve for the interest rate
  - Compute payments to accumulate a future sum
  - Compute payments to amortize a loan

Time Value of Money—Extensions

- Rate of Return or Discount Rate
- Example 3-5:
  - Reed & Portland Trucking is financing a new truck with a loan of $10,000, to be repaid in 5 annual end-of-year installments of $2,504.56. What annual interest rate is the company paying?

Time Value of Money—Extensions

- Set P/Y = 1:

PV = $10,000

n = 5

PMT = ($2504.56)

FV = $0

CPT **i** = 8%

Time Value of Money—Extensions

- Example 3-6:
  - A bank makes a $100,000 loan and will receive payments of $805 each month for 30 years as repayment. What is the rate of return to the bank for making this loan?
  - This is also the cost to the borrower.

Time Value of Money—Extensions

- Set P/Y = 12

PMT = $805

n = 360

PV = ($100,000)

FV = $0

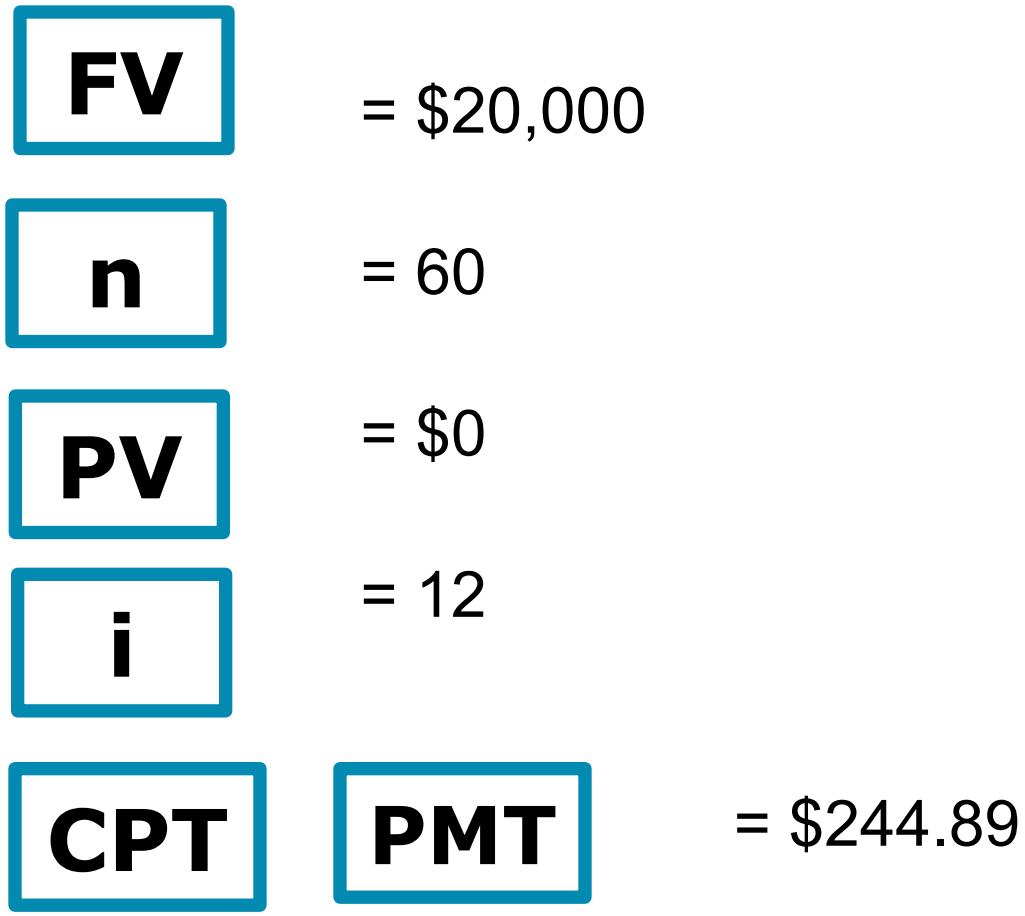
CPT **i** = 9%

Time Value of Money—Extensions

- Example 3-7: Accumulating a Future Sum
  - An individual would like to purchase a home in five (5) years. The individual will accumulate enough money for a $20,000 down payment by making equal monthly payments to an account that is expected to earn 12% annual interest compounded monthly. How much are the equal monthly payments?

Time Value of Money—Extensions

- Set P/Y = 12



Time Value of Money—Extensions

- The Power of Compounding
- In Example 3-7, our saver deposited

  $244.89 × 60 = $14,693.40
- Interest earned was

  $20,000 − $14,693.40 = $5,306.60

Time Value of Money—Extensions

- Example 3-8: Amortizing a Loan
  - Your company would like to borrow $100,000 to purchase a piece of machinery. Assume that you can make one payment at the end of each year, the term is 15 years, and interest rate is 7%. What is the amount of the annual payment?

Time Value of Money—Extensions

- Set P/Y = 1:

PV = $100,000

n = 15

FV = $0

i = 7

CPT **PMT** = $10,979.46

Equivalent Nominal Annual Rate

- ENAR = Equivalent Nominal Annual Rate
- EAY = Effective Annual Yield

$$ENAR = \left[(1 + EAY)^{\frac{1}{m}} - 1 \right] \cdot m$$