

OPSMGT 357

Project Management

Program Evaluation
Review Technique (PERT)

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Today's Learning Objectives

- ▶ Explain the rationale for estimating project work
- ▶ Have a broad understanding of PERT analysis
- ▶ Be able to calculate expected project duration
- ▶ Be able to calculate probability of project completion by specific time
- ▶ Be able to explain when would a PM choose PERT rather than the CP method

Estimating Project Time and Cost

- ▶ Once tasks have been identified (WBS) the time and resource requirements must be determined. This is called estimating.
- ▶ Estimating is the process of forecasting or approximating the time and cost of completing project deliverables. The task of balancing expectations of stakeholders and need for control while the project is implemented.

Description	Duration (Days)
Identify topic	
Research topic	
Draft paper	
Edit paper	
Create graphics	
References	
Final draft	

Factors Influencing the Quality of Estimates

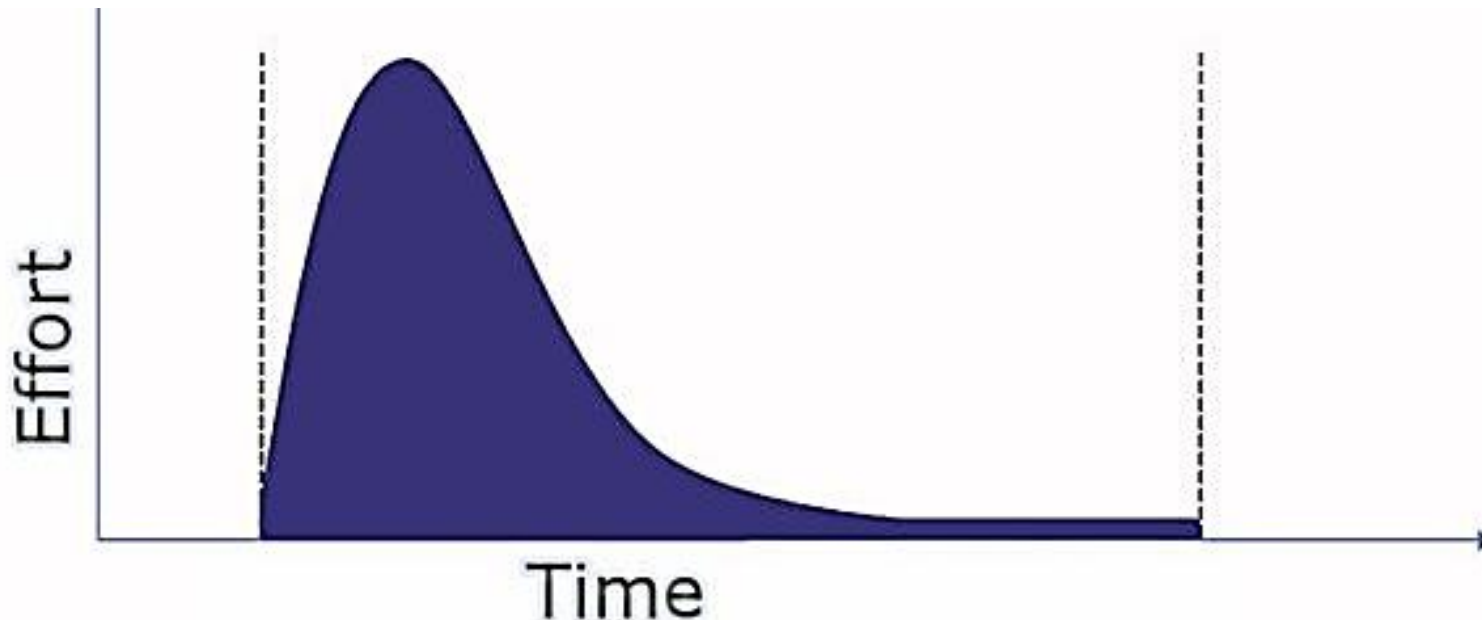
A typical statement in the field is the desire to “have a 95 percent probability of meeting time and cost estimates.”

Past experience is a good starting point for developing time and cost estimates. But past experience estimates must almost always be refined by other considerations to reach the 95 percent probability level. Factors related to the uniqueness of the project will have a strong influence on the accuracy of estimates. Project, people, and external factors all need to be considered to improve quality of estimates for project times and costs.

- ▶ *Planning Horizon*
- ▶ *Project Duration*
- ▶ *People*
- ▶ *Project Structure and Organization*
- ▶ *Parkinson's Law*

Parkinson's Law

- ▶ You can not do a time or cost estimate without considering who will actually perform the task (resource)?
- ▶ You must base the estimate on historical data or mental model
- ▶ **Parkinson's law:** Work expands to fill the time allowed.



Suggestions For Effective Estimations

- ▶ Historical Data: Learn from the past
- ▶ Level of Detail: The smaller the unit of the work the more accurate your estimate is likely to be
- ▶ Ownership of the Estimate: If the individual supplying the estimate owns it, it is likely to be more accurate
- ▶ Human Productivity:

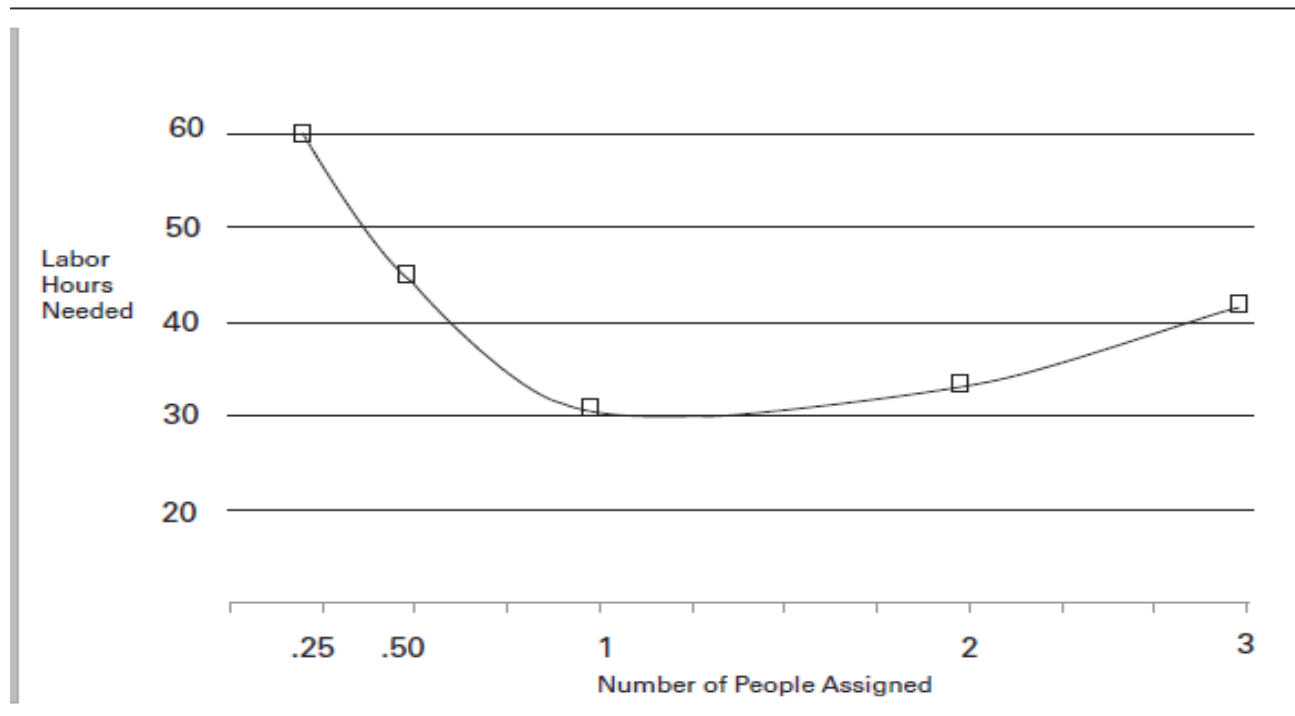
HUMAN PRODUCTIVITY

Productivity Factor	Hours	Cost/Hour	Labor Cost	Duration
Base Estimate	40	\$75	\$3,000	5.00
Project Loss (15%)	6	\$75	\$450	0.75
Rework/Debug (10%)	4	\$75	\$300	0.50
Subtotal (Direct Cost)	50	\$75	\$3,750	6.25
Labor Overhead	6	-	-	0.75
Total For Scheduling	56	-	-	7.00

Human Dynamic, People Are Not Robots

- ▶ What happened as a person's time is split? why?
- ▶ What happened as the number of people increase? Why?

TIME, COST, RESOURCE TRADE-OFF



Program Evaluation Review Technique (PERT)

- ▶ PERT is a project management technique, used to manage uncertain activity times of a project.



Project Evaluation and Review Technique

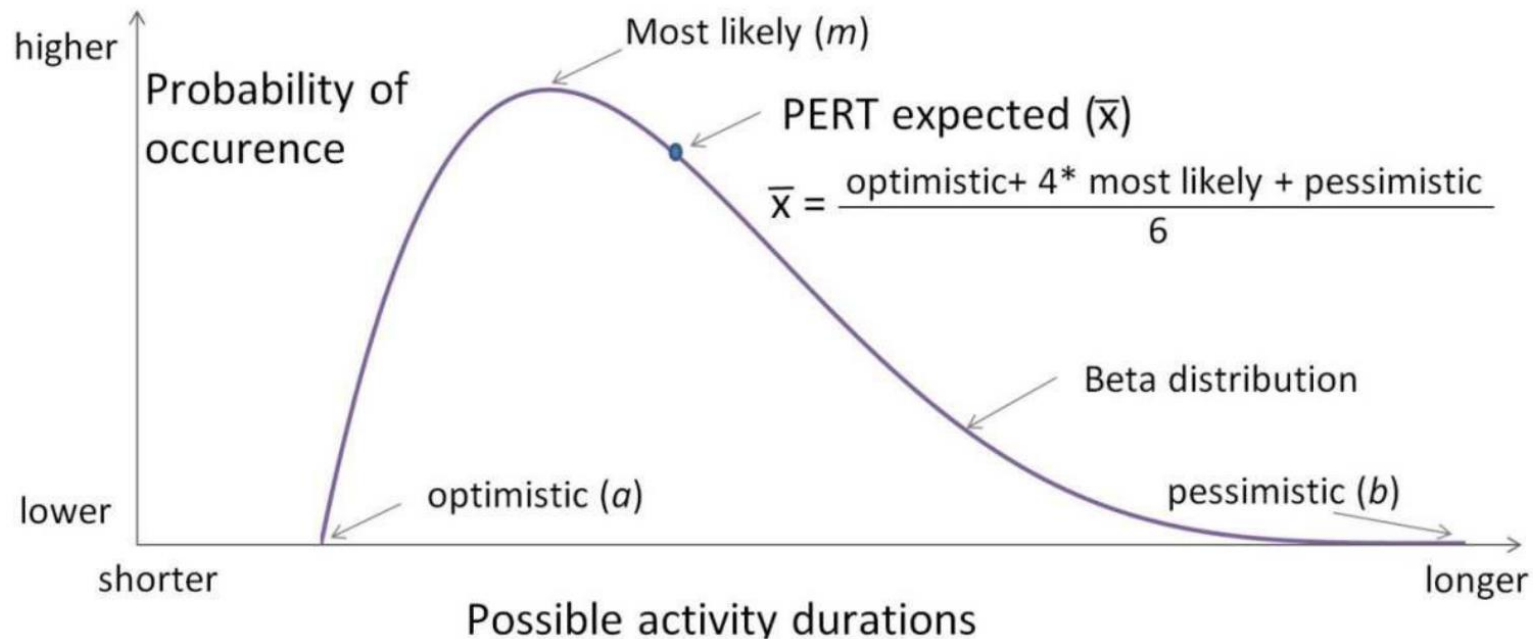
Three Point Estimate Technique

- ▶ Project manager can apply simple formulas to improve estimation accuracy
- ▶ Assume each activity's duration has a range that follows statistical distribution.
- ▶ Basically, this means each activity has three time estimates: **optimistic**, **most likely**, and **pessimistic**



Three Point Estimate Technique

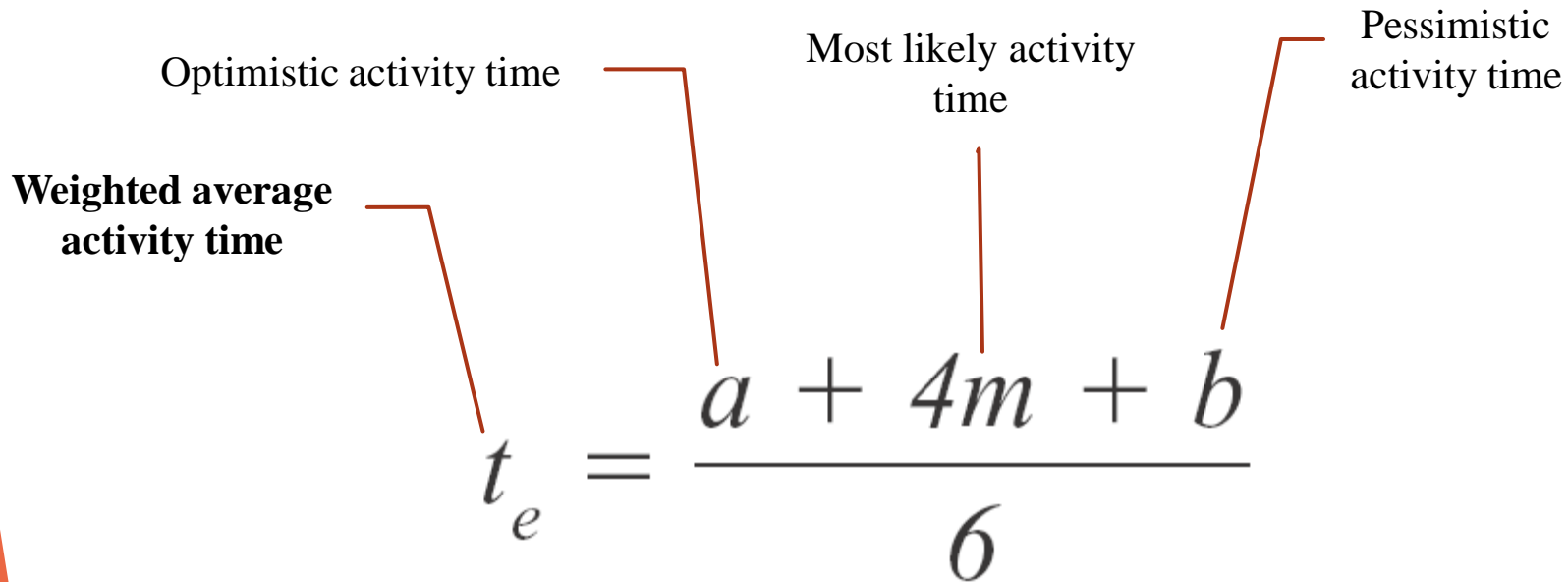
- ▶ Assume each activity's duration has a range that follows statistical distribution.
- ▶ Basically, this means each activity has three time estimates: **Optimistic, Most likely, and Pessimistic**



Program Evaluation Review Technique (PERT)

Weighted average time can be computed for each activity by the following formula:

Each activity has three-time estimates: optimistic, most likely, and pessimistic



Optimistic activity time

Most likely activity time

Pessimistic activity time

Weighted average activity time

$$t_e = \frac{a + 4m + b}{6}$$

Program Evaluation Review Technique (PERT)

The variability in the activity time estimates is approximated by the following equations:

Standard deviation for the **activity**

$$\sigma_{t_e} = \left(\frac{b - a}{6} \right)$$

Pessimistic activity time

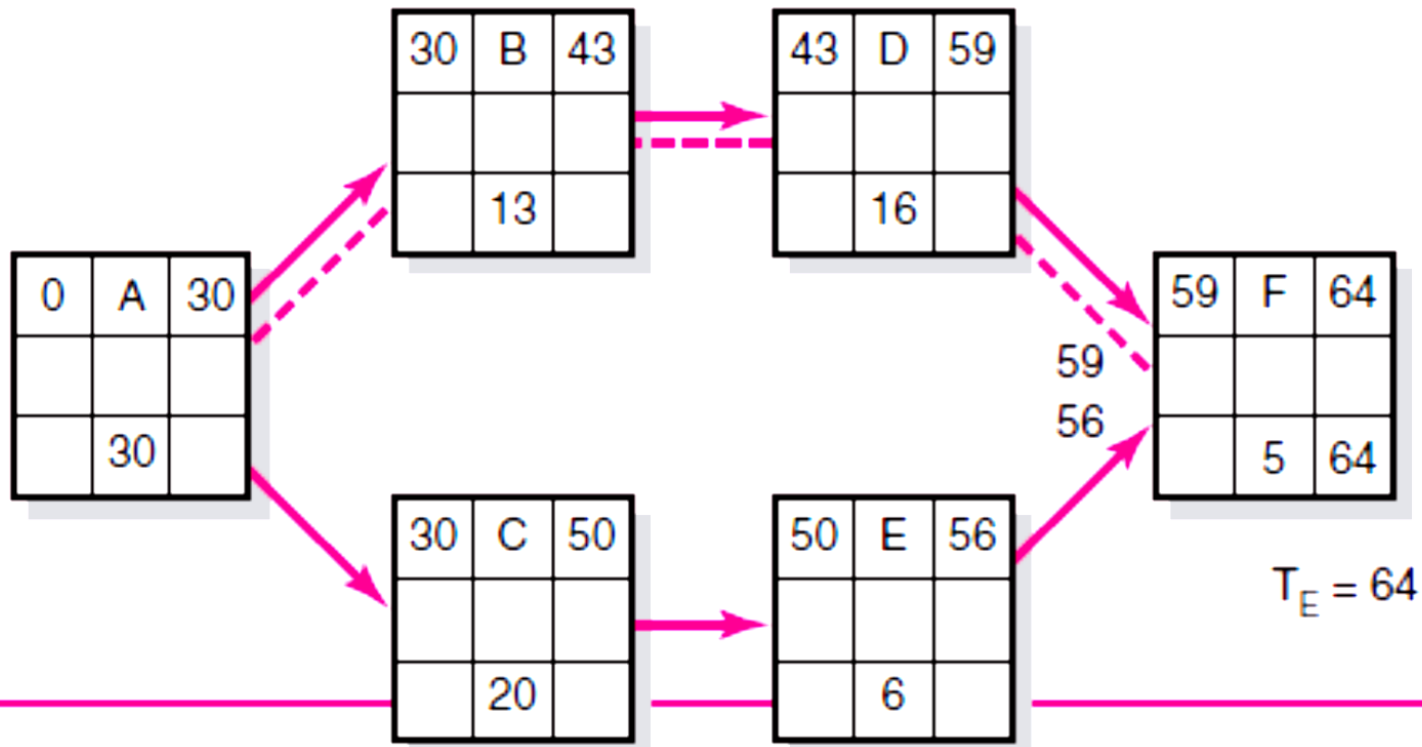
Optimistic activity time

The diagram illustrates the PERT standard deviation formula. A callout line from the text 'Standard deviation for the activity' points to the symbol σ_{t_e} . Another callout line from the text 'Pessimistic activity time' points to the variable b in the numerator of the fraction. A third callout line from the text 'Optimistic activity time' points to the variable a in the numerator. The denominator is the constant 6.

Example: Three Point Estimate Technique

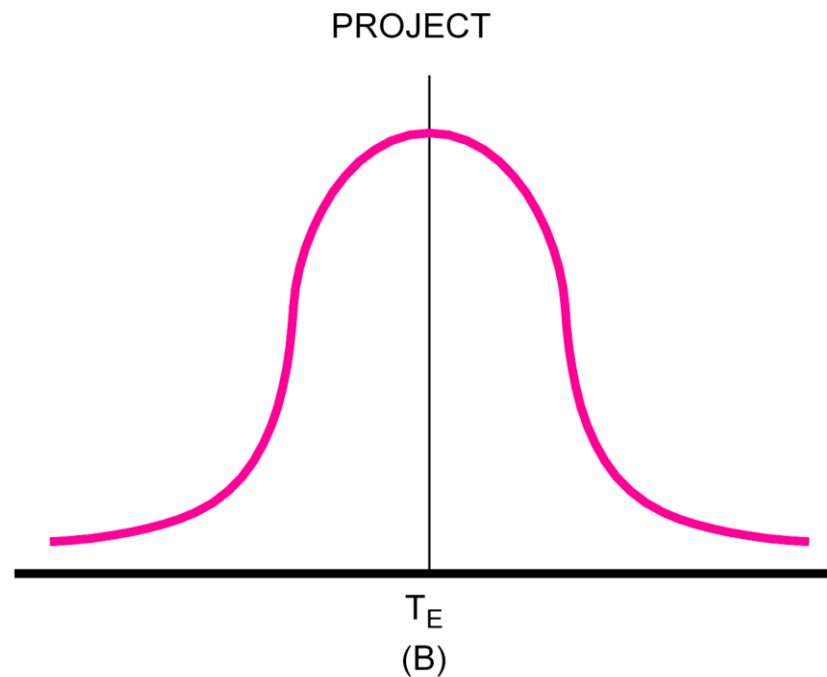
Activity	Predecessor	Optimistic	Most likely	Pessimistic	Weighted ave. duration	Variance
A	-	17	29	47	30	25
B	A	6	12	24	13	9
C	A	16	19	28	20	4
D	B	13	16	19	16	1
E	C	2	5	14	6	4
F	D,E	2	5	8	5	1

Example: Three Point Estimate Technique



Project Distribution Function

Central limit theorem (CLT) establishes that, in many situations, when **independent random variables** are added, their properly normalized sum tends toward a normal distribution (informally a "bell curve") even if the original variables themselves are not normally distributed.



The Average Project Duration

Average project duration (T_E) is the sum of all the average activity times along the critical path (sum of t_e).

The average project duration follows a “Normal” distribution.

$$T_E = \sum t_e$$

Average activity time
along the **critical path**



Project Standard Deviation

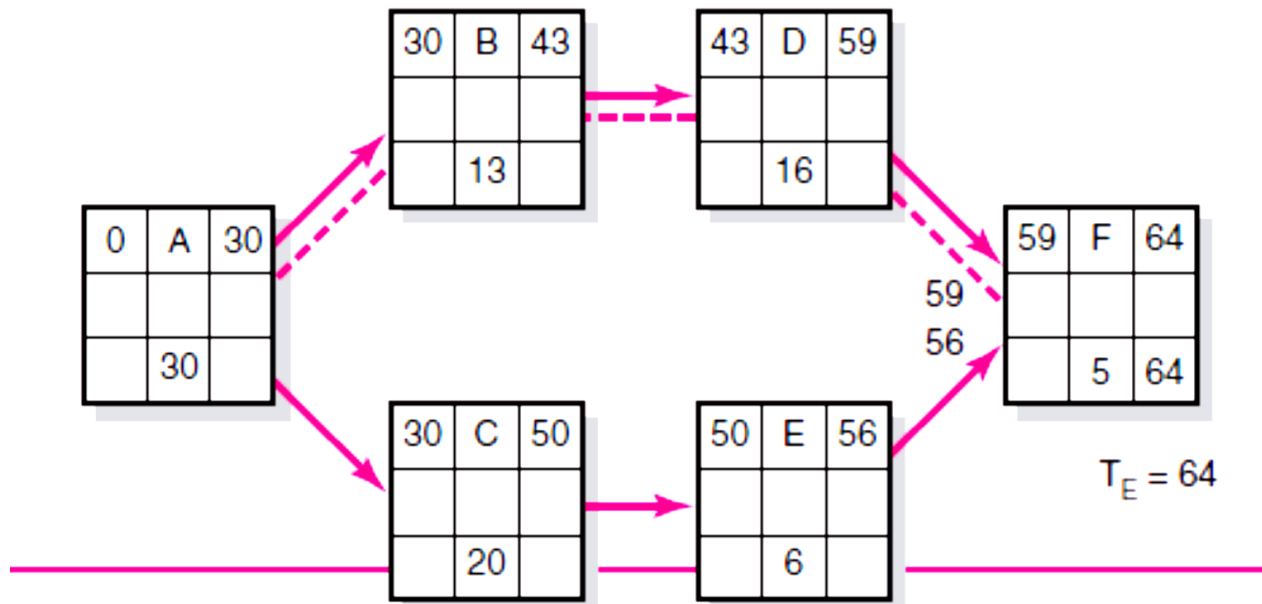
$$\sigma_{T_E} = \sqrt{\sum \sigma_{t_e}^2}$$

Standard deviation
for the **Project**

Squared Standard
deviation for **critical
activities**

Example- Cont.

- ▶ Average project duration: 64
- ▶ Project standard deviation: $\sqrt{25 + 9 + 1 + 1} = 6$



Program Evaluation Review Technique (PERT)

Now we know:

- Average project duration (T_E)
- Standard deviation for the project (σ_{T_E})

Knowing these two allow us to compute the probability of completing the project by a specific time- using standard statistical tables or Excel to calculate the probability

$$Z = \frac{T_S - T_E}{\sqrt{\sum \sigma_{t_e}^2}}$$

Scheduled project duration

Critical path duration = average project duration

Probability (of meeting scheduled duration)
found in statistical table/ by Excel

Example- Cont.

- ▶ What is the probability the project will be completed before a scheduled time (TS) of 67?

$$\begin{aligned} Z &= \frac{T_S - T_E}{\sqrt{\sum \sigma_{t_e}^2}} \\ &= \frac{67 - 64}{\sqrt{25 + 9 + 1 + 1}} \\ &= \frac{+3}{\sqrt{36}} \\ &= +0.50 \\ P &= 0.69 \end{aligned}$$

z	0.00	0.01	0.02	0.03	
0.0	0.5000	0.5040	0.5080	0.5120	0
0.1	0.5398	0.5438	0.5478	0.5517	0
0.2	0.5793	0.5832	0.5871	0.5910	0
0.3	0.6179	0.6217	0.6255	0.6293	0
0.4	0.6554	0.6591	0.6628	0.6664	0
0.5	0.6915	0.6950	0.6985	0.7019	0
0.6	0.7257	0.7291	0.7324	0.7357	0
0.7	0.7580	0.7611	0.7642	0.7673	0
0.8	0.7881	0.7910	0.7939	0.7967	0
0.9	0.8159	0.8186	0.8212	0.8238	0
1.0	0.8413	0.8438	0.8461	0.8485	0
1.1	0.8643	0.8665	0.8686	0.8708	0
1.2	0.8849	0.8869	0.8888	0.8907	0
1.3	0.9032	0.9049	0.9066	0.9082	0
1.4	0.9192	0.9207	0.9222	0.9236	0

Exercise 1: Group assignment project

► Project: Group assignment

You have three estimated time for each activity, Find the probability of completing project in 14 days

ID	Description	Predecessors	Optimistic duration	Most likely duration	Pessimistic duration
A	Identify Topic	-	1	2	4
B	Research topic	A	3	5	8
C	Draft Paper	B	2	3	5
D	Edit Paper	C	1	2	3
E	Create graphics	C	1	2	4
F	References	C	1	2	3
G	Final draft	D,E,F	1	2	4

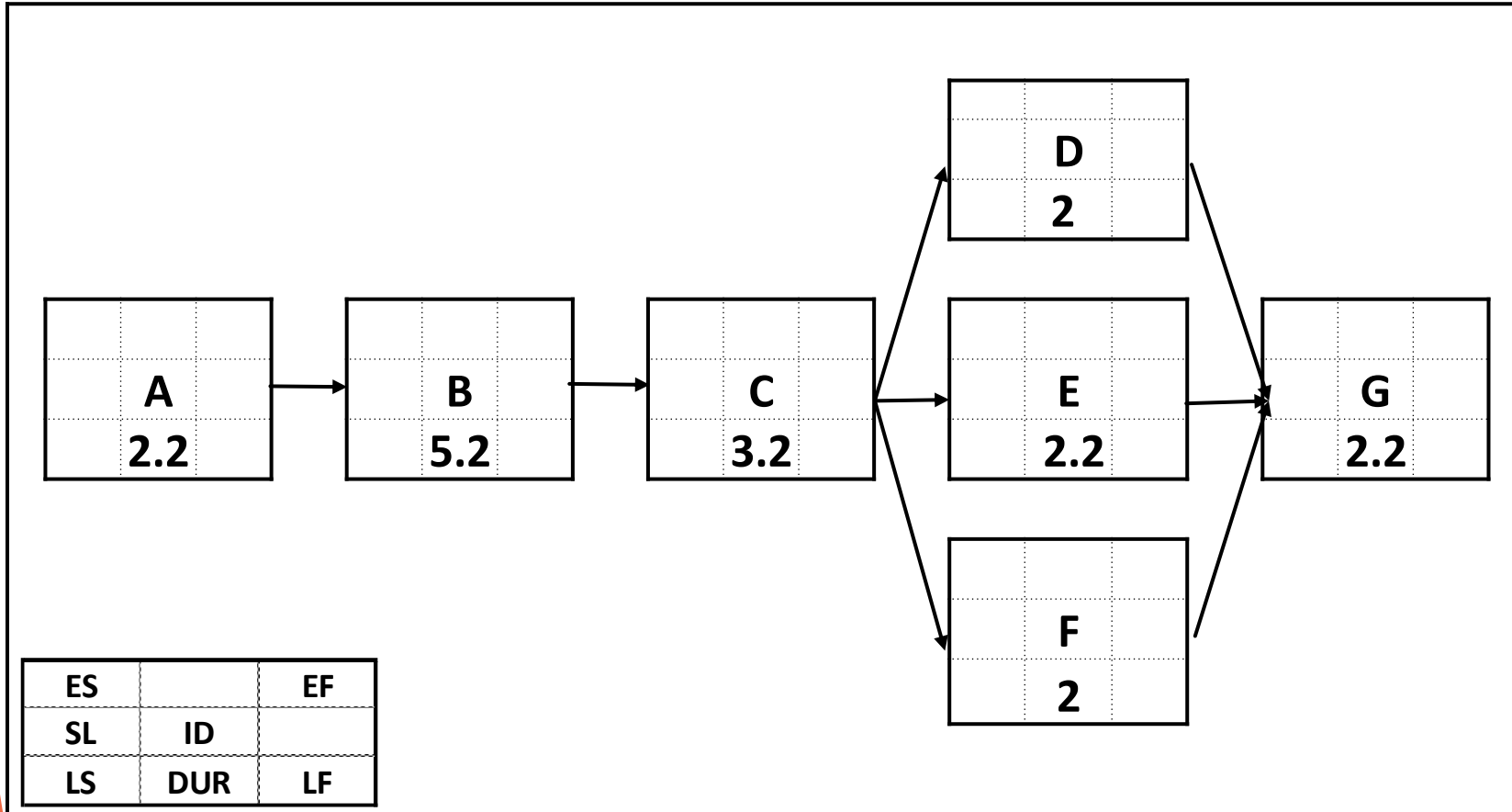
Exercise 1: Group assignment project

1. Find weighted average time and variance for each activity
2. Find critical path based on weighted average time
3. Find the average project duration (critical path duration)
4. Find the project standard deviation
5. Find the probability of completing project in 14 days

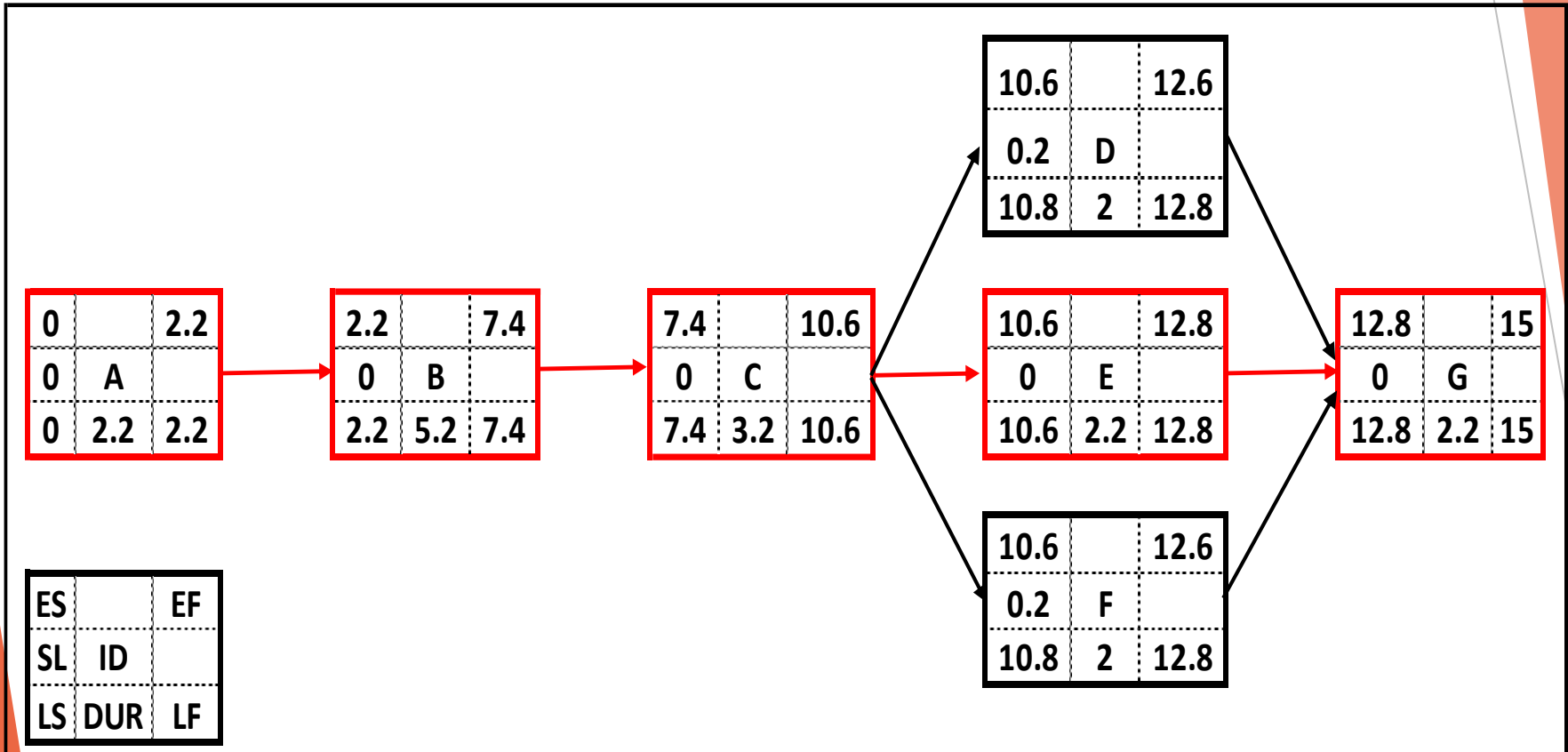
1. Find weighted average time and variance for each activity

ID	Description	Predecessors	a	m	b	te	σ_{te}^2
A	Identify Topic	-	1	2	4	2.2	0.25
B	Research topic	A	3	5	8	5.2	0.69
C	Draft Paper	B	2	3	5	3.2	0.25
D	Edit Paper	C	1	2	3	2.0	0.11
E	Create graphics	C	1	2	4	2.2	0.25
F	References	C	1	2	3	2.0	0.11
G	Final draft	D,E,F	1	2	4	2.2	0.25

2. Find critical path based on weighted average time



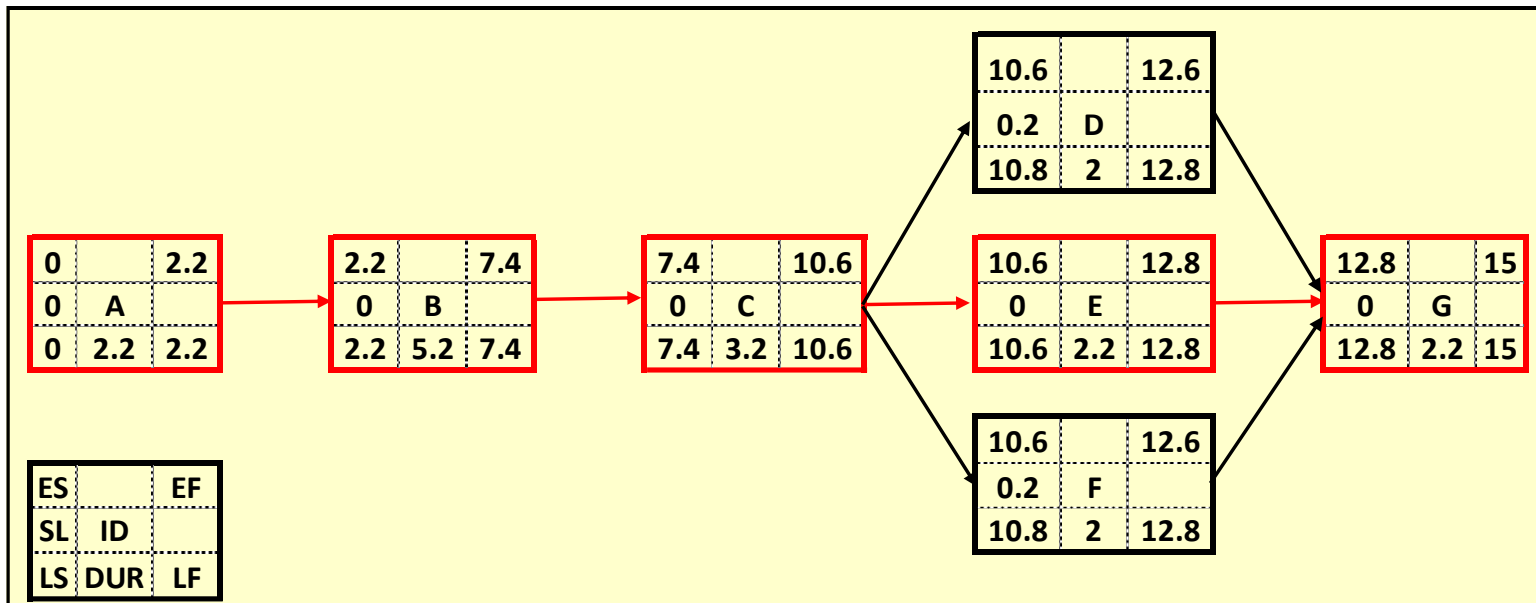
2. Find critical path based on weighted average time



3. Find the average project duration (critical path duration)

Average **project** duration:

$$T_E = 2.2 + 5.2 + 3.2 + 2.2 + 2.2 = 15$$

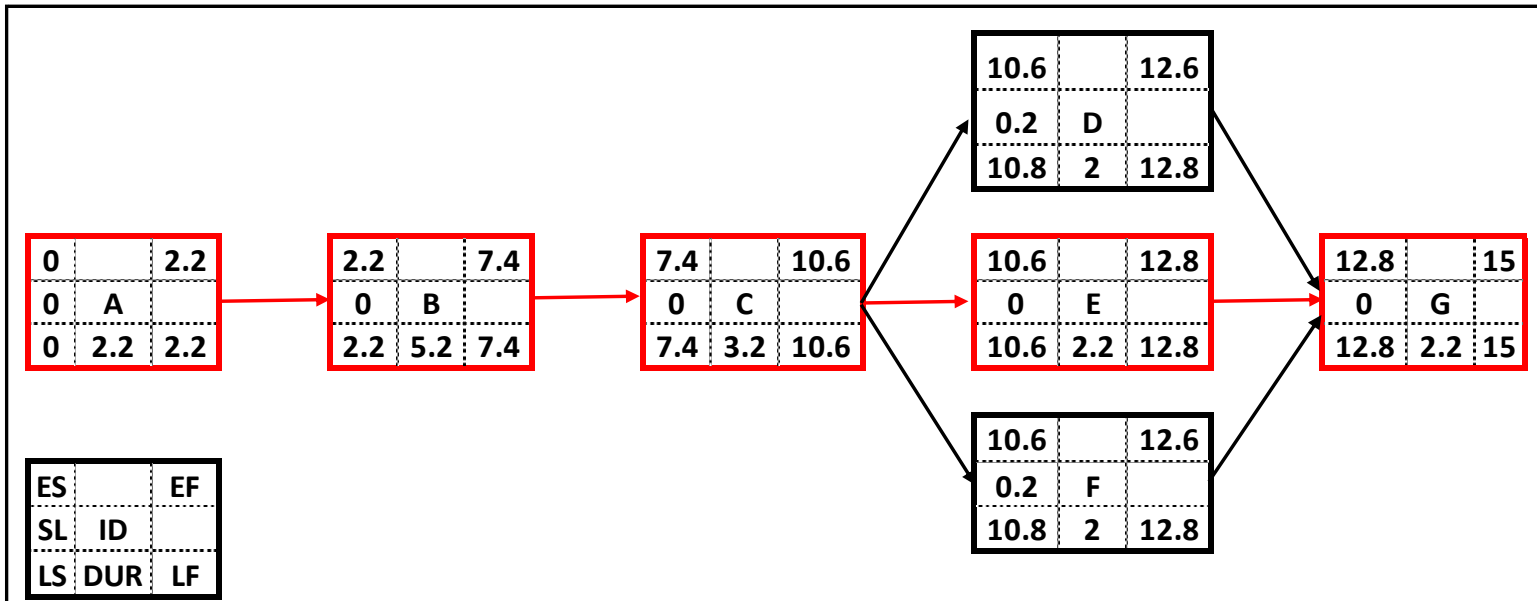


4. Find the project standard deviation

SD of the project:

$$\sigma_{T_E} = \sqrt{\sum \sigma_{t_e}^2}$$

$$\sigma_{T_E} = \sqrt{0.25 + 0.69 + 0.25 + 0.25 + 0.25} = 1.3$$



5. Find the probability of completing project in 14 days

Average project duration	$2.2+5.2+3.2+2.2+2.2= 15$
Project SD of the	$\sqrt{0.25 + 0.69 + 0.25 + 0.25 + 0.25} = 1.3$
$Z = \frac{T_S - T_E}{\sqrt{\sum \sigma_{t_e}^2}}$	$Z=(14-15)/1.3= - 0.64$

➤ **Use statistical table**

$$\varphi(-x)=1-\varphi(x)$$

$$\varphi(-0.64)=1-\varphi(0.64)$$

➤ **Use Excel**

Statistical Table

Z=0.64

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5	0.504	0.508	0.512	0.516	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.591	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.648	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.67	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.695	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.719	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.758	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.791	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.834	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.877	0.879	0.881	0.883
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.898	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.975	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.985	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.992	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.994	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.996	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.997	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.998	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.999	0.999

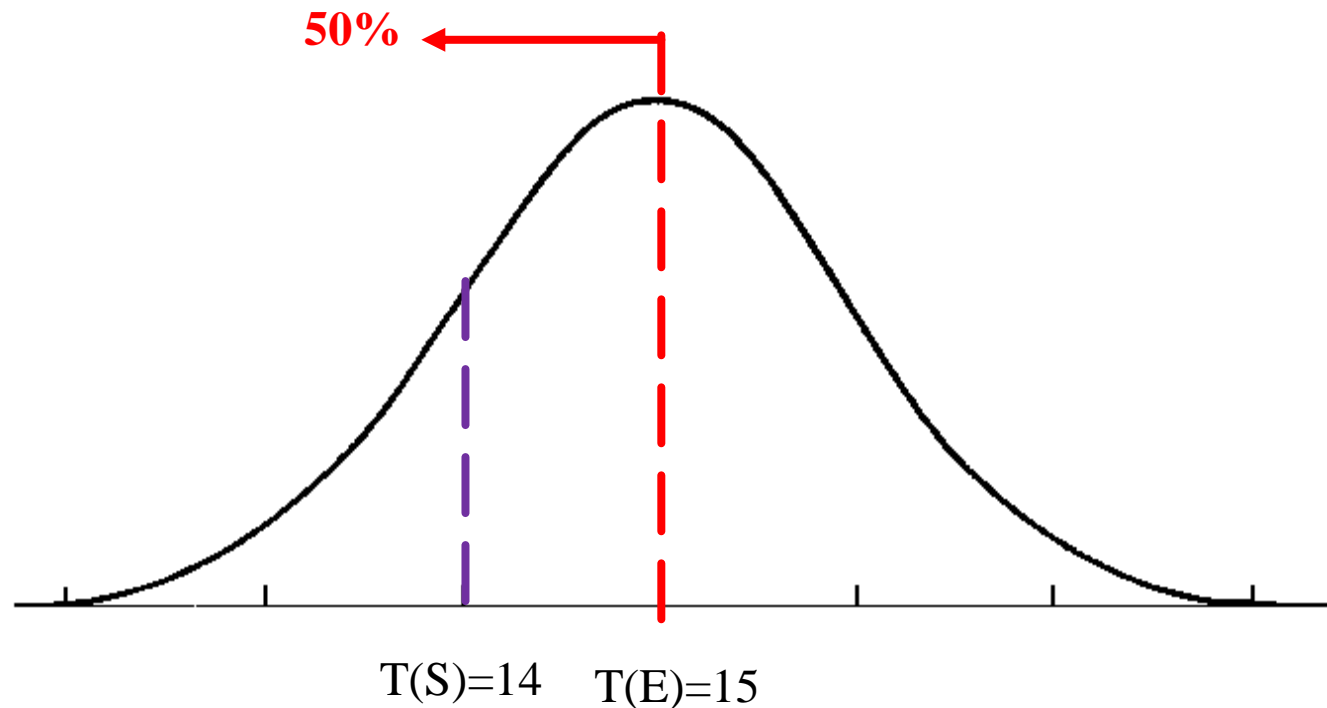
5. Find the probability of completing project in 14 days

$$\varphi(-0.64) = 1 - \varphi(0.64)$$

$$\varphi(0.64) = 0.7389$$

$$\varphi(-0.64) = 1 - 0.7389 = \mathbf{0.2611}$$

We have **26.11%** chance of completing the project within 14 days



Excel

Use this formula in excel:

`=NORM.S.DIST(Z, Cumulative)`

**Select the cell
that you
calculate Z**

True

CPM vs. PERT

- ▶ PERT uses probability – CPM does not
- ▶ CPM allows us to ask, “How many days (on average) is the longest uninterrupted path through the project?”
- ▶ PERT allows us to ask, “How likely is it that we complete this project after XX days?”
- ▶ CPM needs ONE point of duration estimating for each task-
Reasonable time estimate
- ▶ PERT needs THREE points of duration estimating for each task
(shortest – most likely – longest)- High precision time estimate

Exercise 2: PERT

The activity times and predecessors are given in below table

ID	Predecessors	Optimistic duration	Most likely duration	Pessimistic duration
A	-	6	12	24
B	A	16	19	28
C	A	4	7	10
D	B	21	30	39
E	C	17	29	47
F	D,E	13	16	19

Exercise 2: PERT

1. What is the expected project duration?
2. What is the probability of completing the project by day 75? within 85 days?
3. How many extra days would you require for a 94.95% probability of completion?
4. When would a PM choose PERT rather than the CP method (CPM)?

Today's Learning Objectives

- ▶ Explain the rationale for estimating project work
- ▶ Have a broad understanding of PERT
- ▶ Be able to calculate expected project duration
- ▶ Be able to calculate probability of project completion by specific time
- ▶ Be able to explain when would a PM choose PERT rather than the CP method