PART II

ECON208 - Macroeconomic Analysis - Final Exam

2h30 to complete the exam + 30 minutes to upload answers

Candidates should answer TWO questions from Section A and TWO questions from Section B. All questions carry equal marks. Please answer each section on a different booklet. Use of non-programmable calculators is allowed.

Exam Guidelines

You have 2.5 hours to complete this paper. Within this time, you must answer and prepare a single response in digital PDF format (typing all large blocks of text and scanning of any formula, diagrams, graphs etc.), and most importantly, you must save your file before the end of the 2.5 hour window (evidenced in the file properties date/time). You then have a further 30 minutes in which to upload your examination file to the Moodle page. If you have any issues uploading your file, be sure to e-mail your file to `UG_economics@lancaster.ac.uk` *before the end of the upload window to avoid penalty.*

Section A

1. Consider the Solow model.

   (a) Define the concept of "Golden Rule" level of capital. How it is possible to identify it?

   (b) Using suitable diagrams, compare the different dynamics for the levels and growth rates of capital and output following:

      i. a new wave of immigration

      ii. an increase in the saving rate

      iii. a one-shot foreign investment which increase the size of the available stock of capital

   Answer: The golden rule level of capital is that level of capital that would guarantee the maximum consumption, this can be found where the slope of the production funcion coincides with the slope of the break even investment line. Using graphs only show that a wave of immigration pushes capital per capita at a lower steady state and the growth rate is decreasing the closer we get to the new steady state. Increasing the saving rate would increase capital and output, foreign investment would increase the capital stock but just temporary, bringing over time the economy back to the original steady state.

2. Consider the AD-AS model:

   (a) Explain with your own words what is the Phillips Curve.

   (b) Starting from the price and wage setting equations derive the Phillips Curve.

   (c) Explain the role of expectations in the Phillips Curve. Compare the cases of adaptive expectations and rational expectations and explain what are the implications for the monetary policy maker.

3. Consider an economy characterised by:

$$C = 500 + 0.8(Y - T)$$

$$I = 400 - 120r$$

$$G = 300$$

$$T = 0.25Y$$

$$L(r, Y) = Y - 300r$$

$$\frac{M}{P} = 600$$

   where C, Y, I, G, T, r, L and $\frac{M}{P}$, denote consumption, output, investment, government spending, taxes, the interest rate, liquidity preferences and the real money supply, respectively.

   (a) Derive expressions for the IS and the LM and plot the two curves.

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(b) Find the equilibrium interest rate and the equilibrium level of income.

(c) Derive the Keynesian multiplier and comment. Does it change with taxation?

(d) Calculate and interpret the effects on Y and r of an increase of money supply that bring $\frac{M}{P}$ to 1200.

(e) Derive an expression for the Aggregate Demand given the model in a).

Answer:

a. IS: Y=3000-300r LM: Y=600+300r

b. In equilibrium r=4 and Y=1800

c. The Keynesian multiplier reads as 1/(1-0.8(1-0.25))=1/0.4=2.5. In this case the taxation proportional to income reduces the power of the Keynesian multiplier (compared to the case of lump-sum taxation). This because any increase in income is taxed proportionally and therefore limits the response of consumption.

d. IS: Y=3000-300r LM: Y=1200+300r r=3 Y=2100 The monetary expansion has generated an excess of money that triggered a fall in the interest rate. This resulted in an increase in investment and an increase in output (via a multiplicative effect)

e. substitute LM into IS via the interest rate. Collect Y.

4. Consider the AS-AD model:

(a) show, using a suitable graph, the short run and medium run effects on output, prices and the interest rate of a positive technology shock. Explain your answers.

(b) Define and compute the natural level of unemployment.

(c) What are the possible policies that can push higher the level of GDP in the medium run?

Answer: students should mention that the positive technology shock would shift the natural level of output to the right this might imply further reduction in prices. to compute the natural level of unemployment just equate price and expected price in the AS function. Policies to promote growth in the medium term are similar to those who promote log run growth, so invetments in research and development above all.

Section B

1. **Capitalists and Financial Openness**

We consider an economy with no population growth, i.e., $n = 0$, which produces a final good according to a technology of production described by

$$y = Ak^{\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where y is output per capita, k the stock of capital per capita, A measures the level of technology. We denote the capital depreciation rate by δ. There are capitalists and workers. Workers do not save while capitalists save a fraction β of after-tax capital income. The government finance government spending by levying a tax $0 < x < 1$ on capital income Rk. Total taxes T are thus equal to: $T = xRk$ where $R = r + \delta$. R is a return for capitalists and a cost for firms. Since $Rk = \alpha y$, savings per capita is $\beta(1-x)\alpha y$.

(a) Determine the capital stock per capita in a closed economy economy, k^c, by using the capital market equilibrium (hint: investment per capita is δk). Next, determine the capital cost in closed economy, R^c, which is equal to the marginal product of capital for $k = k^c$.

Answer: By equating investment to savings per capita, we get the capital stock per capital in a closed economy:

$$\begin{aligned} \delta k &= \beta\alpha(1-x)y, \\ \delta k &= \beta\alpha(1-x)Ak^{\alpha}, \\ k &= \left[\frac{\beta\alpha(1-x)A}{\delta}\right]^{\frac{1}{1-\alpha}}. \end{aligned} \tag{2}$$

The capital cost is:

$$\begin{aligned} R^c &= \alpha A(k^c)^{\alpha-1}, \\ &= \alpha A\frac{\delta}{\beta\alpha(1-x)A} \\ R^c &= \frac{\delta}{\beta(1-x)}. \end{aligned} \tag{3}$$

(b) Derive the capital stock per capita in an open economy economy, $k^{\star}$, by equating the marginal product of capital to the capital cost $R^{\star} = r^{\star} + \delta$.

Answer: The MPK is $\frac{\partial y}{\partial k} = \alpha Ak^{\alpha-1}$. The economy chooses an optimal capital stock per capita by equating the MPK to the capital cost in open economy:

$$\begin{aligned} \alpha Ak^{\alpha-1} &= R^{\star}, \\ k^{\star} &= \left(\frac{\alpha A}{R^{\star}}\right)^{\frac{1}{1-\alpha}}. \end{aligned} \tag{4}$$

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(c) The net capital flows in percentage of GDP, d, are $d = s - \delta \frac{k^\star}{y^\star}$ where $s = \beta(1-x)\alpha$ is the rate of savings. Determine d by inserting $k^\star$ (and using (1)) and next determine the condition (which involves R^c and $R^\star$) such that $d < 0$.

Answer: We first calculate the investment rate:

$$
\begin{aligned}
\delta \frac{k^\star}{y^\star} &= \delta \frac{(k^\star)^{\alpha-1}}{A}, \\
k^\star &= \frac{\delta}{A}\frac{A\alpha}{R^\star} = \frac{\alpha\delta}{R^\star}. \qquad (5)
\end{aligned}
$$

Net capital flows thus read: $d = \alpha\left[\beta(1-x) - \frac{\delta}{R^\star}\right]$. We have $d < 0$ when $R^\star < \frac{\delta}{\beta(1-x)} = R^c$ where we made use of (3).

2. **Shifts in Foreign Demand: Floating vs. Fixed Exchange Rates**

Let us consider an open economy which has removed all barriers to capital mobility. The open economy comprises three agents: households, firms and monetary authorities. We denote GDP by Y, the domestic interest rate by r, the exchange rate (i.e., the pound sterling price of one euro) by e (a rise in e means a depreciation of the pound sterling), and the world interest rate by $r^\star$. Prices are fixed and normalized to unity. The money market is described by $L(r,Y) = M^S$ where money supply $M^S = \bar{M} + H$ includes an exogenous component, $\bar{M}$, and an endogenous component $H = \Delta\text{RES}$ which is the change in foreign currency reserves. The goods market is described by $Y = C(Y) + I(r) + NX(Y, Y^\star, e)$ where C is households' final consumption expenditure, I investment, NX net exports. The new variable is $Y^\star$ which represents world GDP. The behavior of households and firms are characterized by the following reaction functions: $0 < C_Y < 1$, $I_r < 0$, $NX_e > 0$, $NX_Y < 0$, $L_r < 0$, $L_Y > 0$. We assume that exports are increasing in world GDP, $Y^\star$, and thus net exports rise when $Y^\star$ increases, i.e., $NX_{Y^\star} > 0$.

(a) Show graphically the effects of a decline in world GDP, $Y^\star$, in a fixed exchange rate regime ($H \neq 0$) in the (Y, r)-space. Explain the adjustment from the initial to the new macroeconomic equilibrium.

Answer. A decline in world output lowers net exports and thus shifts the IS-schedule to the right. Lower economic activity puts downward pressure on the domestic interest rate which makes domestic assets less attractive and leads to a capital outflow. The counterpart of excess supply for the domestic currency is a decline in the stock of international reserves in a fixed exchange rate regime whilst money supply falls. Thus the LM-schedule shifts to the left as well which amplifies the recessionary effect on economic activity caused by lower net exports.

(b) Derive the change in domestic output, Y, and in foreign currency reserves, H, caused by a decline in $Y^\star$, i.e., $dY/dY^\star$ and $dH/dY^\star$, in a fixed exchange rate system. Next, determine the devaluation of the domestic currency, $d\bar{e}/dY^\star$ which is necessary to keep the economic activity unchanged, i.e., $dY = 0$.

Answer. In a fixed exchange rate regime, the exchange rate becomes an exogenous variable, i.e., $e = \bar{e}$, while the change in the stock of international reserves is an endogenous variable. Like previously, perfect capital mobility implies that the domestic interest rate must stick to the foreign interest rate, $r^\star$.Differentiating the goods market and the money market equilibrium leads to:

$$dY\left(1 - C_Y - NX_Y\right) = NX_e d\bar{e} + NX_{Y^\star} dY^\star, \tag{6a}$$
$$L_Y dY = d\bar{M} + dH. \tag{6b}$$

Keeping first the exchange rate fixed, i.e., setting $d\bar{e} = 0$, the GME (6a) can be solved for the change in output, i.e., $\frac{dY}{dY^\star} = \frac{NX_{Y^\star}}{1-C_Y-NX_Y} < 0$. Since $d\bar{M} = 0$, and plugging $dY/dY^\star$ into (6b) leads to the change in foreign currency reserves, i.e., $\frac{dH}{da} = L_Y \frac{NX_{Y^\star}}{1-C_Y-NX_Y} < 0$. To determine the magnitude of devaluation which is necessary to keep $dY = 0$, we solve (6a) for $d\bar{e}$ while setting $dY = 0$, i.e., $\frac{d\bar{e}}{dY^\star} = -\frac{NX_{Y^\star}}{NX_e} > 0$.

(c) Derive the change in domestic output, Y, and in the exchange rate, e, caused by a decline in $Y^\star$, i.e., $dY/dY^\star$ and $de/dY^\star$, in a floating exchange rate system ($H = 0$). Explain the adjustment from the initial to the new macroeconomic equilibrium.

Answer. Differentiating the goods market and the money market equilibrium leads to:

$$dY\left(1 - C_Y - NX_Y\right) = NX_e de + NX_{Y^\star} dY^\star, \tag{7a}$$
$$L_Y dY = d\bar{M} + dH. \tag{7b}$$

Since $dH = 0$ in a floating exchange rate regime and money supply does not change, i.e., $d\bar{M} = 0$, the MME (7b) implies that $dY = 0$. Because output is unchanged, setting $dY = 0$, the GME (7a) can be solved for the change in the exchange rate, i.e., $de = -\frac{NX_{Y^\star}}{1-C_Y-NX_Y} dY^\star > 0$ because $dY^\star < 0$. .

A fall in world output lowers net exports and thus shifts the IS-schedule to the left. Lower economic activity puts downward pressure on the domestic interest rates which makes domestic assets more attractive and leads to a capital outflow. The excess supply for the domestic currency leads to a depreciation in the exchange rate which exerts a positive impact on net exports that bring the IS-schedule back to its initial position. As a result, in a floating exchange rate regime, any demand shock on the goods market has no effect on economic activity.

3. **Optimism and Over-Borrowing**.

Consider a small open endowment economy without a government that is inhabited by a representative consumer who lives two periods indexed by '1' and '2', respectively. The representative consumer receives exogenously a revenue of Y in period 1 and anticipates a revenue of $Y(1+g)$ in period 2 where g is the expected rate of future growth. The household consumes C_1 in period 1 and C_2 in period 2. Her/his lifetime utility is given by:

$$\Lambda = C_1 C_2. \tag{8}$$

To transfer consumption across time, the household holds a stock of net foreign assets B_1 in period 1. When $B_1 < 0$ ($B_1 > 0$), the country borrows (lends) from (to) abroad. We impose $B_2 = 0$. Period 1 and period 2 budget constraints are given by, respectively:

$$Y = C_1 + B_1, \quad C_2 = R^\star B_1 + Y(1+g), \tag{9}$$

where $R^\star = 1 + r^\star$ with $r^\star$ the (exogenous) world interest rate.

(a) Derive first the intertemporal budget constraint and the optimal trade-off between C_1 and C_2. Then derive optimal consumption in period 1, C_1, and the net international investment position, B_1, which must be both expressed in terms of Y, $R^\star$, g.

Answer. Eliminating B_1 from the first budget constraint by using the second budget constraint, i.e., $B_1 = \frac{C_2 - Y(1+g)}{R^\star}$ leads to the IBC:

$$C_1 + \frac{C_2}{R^\star} = Y + \frac{Y(1+g)}{(R^\star} \equiv \Omega. \tag{10}$$

The intertemporal MRS is computed as the ratio of the marginal utility in period 1, $\frac{dC_1C_2}{dC_1} = C_2$ to the marginal utility in period 2, $\frac{dC_1C_2}{dC_2} = C_1$. The intertemporal MRS must be equal to the return on savings, i.e., $\frac{C_2}{C_1} = R^\star$. Inserting the latter equality, i.e., $\frac{C_2}{R^\star} = C_1$, into the IBC (10) and solving for $C_1^\star$ leads to:

$$\begin{aligned} C_1 + C_1 &= Y + \frac{Y(1+g)}{R^\star}, \\ C_1 &= \frac{Y}{2}\left[1 + \frac{1+g}{R^\star}\right]. \end{aligned} \tag{11}$$

To find the optimal NIIP, we plug (11) into the period 1-budget constraint (first equality in (9)) which leads to:

$$\begin{aligned} B_1 &= Y - C_1, \\ &= Y - \frac{Y}{2}\left(1 + \frac{1+g}{R^\star}\right), \\ &= \frac{Y}{2}\left(\frac{R^\star - 1 - g}{R^\star}\right). \end{aligned} \tag{12}$$

(b) We assume that $1 + g = 3R^\star$. Determine B_1 and explain why the country borrows from abroad.

Plugging $1 + g = 3R^\star$ into eq. (12) leads to:

$$\begin{aligned} B_1 &= \frac{Y}{2}\left(\frac{R^\star - 3R^\star}{R^\star}\right), \\ &= -\frac{Y}{2}2 = -Y. \end{aligned} \tag{13}$$

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Because the country expects a high economic growth and the representative households wishes to smooth consumption over time, it is optimal to borrow from aborad to sustain a high level of consumption across time, i.e., both in the present and in the future.

(c) We now assume that the country was too optimistic about economic growth prospects. While the rate of growth was initially expected to be $g = 3R^{\star} - 1$, the actual rate of growth is $g' = R^{\star} - 1$. By using your answer to question (a), determine first the optimal level of C_2 when $g = 3R^{\star} - 1$ and next when $g' = R^{\star} - 1$. Explain the consequences of overestimating in period 1 the rate of growth in period 2.

Answer. From the optimal trade-off between C_2 and C_1, we have $\frac{C_2}{C_1} = R^{\star}$ and thus $C_2 = R^{\star} C_1$. Plugging $C_1 = \frac{Y}{2}\left[1 + \frac{1+g}{R^{\star}}\right]$ leads to:

$$C_2 = R^{\star} C_1 = R^{\star} \frac{Y}{2}\left[1 + \frac{1+g}{R^{\star}}\right]. \tag{14}$$

When we set $1 + g = 3R^{\star}$, the representative household consumes $C_2 = 2R^{\star}Y$. When we set $1 + g' = R^{\star}$, the representative household must reduce dramatically her/his consumption (by a factor of two) at $C_2' = R^{\star}Y$. Intuitively, households are forward-looking and borrowed from abroad on the basis of expectations of high prospects for her/his future income. At the beginning of period 2, the household has already borrowed $B_1 = -Y$ and when the household learns about the lower actual rate of growth in period 2, he/she must reduce dramatically consumption such that $C_2 = R^{\star} B_1 + Y (1 + g')$.

4. **Taxation and Labor Supply**

Let us consider a closed economy with one final good whose price is normalized to one, i.e., $P = 1$. The representative household has an available time normalized to 1 which can be allocated to work N^S or leisure time l, i.e., $N^S + l = 1$. Each agent derives utility from consumption C and leisure time l:

$$\Lambda \equiv C + \ln l, \quad \gamma > 0. \tag{15}$$

The household supplies labor in exchange of a hourly real wage rate denoted by ω which is taxed at a rate τ. There is no savings. The budget constraint is:

$$C = \omega (1 - \tau) N^S. \tag{16}$$

The representative firm produces an amount Y of the final good by using labor, N:

$$Y = \ln N. \tag{17}$$

(a) Derive labor supply, N^S. Derive labor demand, N^D. Determine equilibrium hours worked, $N^{\star}$.

Answer. To derive the optimal labor supply, eliminate C and l from (15) by using (16) and the time constraint, i.e., $l = -N^S$, to express utility in terms of labor supply:

$$\Lambda = \omega \,.\, (1-\tau) \ .N^S + \ln\left(1-N^S\right).$$

Optimal N^S is reached when $\frac{d\Lambda}{dN^S}=0$. Differentiating Λ w.r.t. N^S and setting the derivative to zero leads to:

$$\begin{aligned} \frac{d\Lambda}{dN^S} &= \omega \,.\, (1-\tau) - \frac{1}{1-N^S} = 0, \\ \omega\,(1-\tau) &= \frac{1}{(1-N^S)}. \end{aligned} \tag{18}$$

Labor demand is determined by the equality between ω and the marginal product of labor (MPL), the latter being equal to $\frac{\partial Y}{\partial N} = \frac{1}{N^D}$. Eliminating ω from labor supply (18) by using $\omega = \frac{1}{N}$ and solving leads to:

$$\begin{aligned} \frac{1}{1-N} &= \frac{1}{N}\,(1-\tau)\,, \\ N &= (1-N)\,(1-\tau)\,, \\ N^{\star} &= \frac{1-\tau}{2-\tau}. \end{aligned} \tag{19}$$

(b) Plot the labor supply- (LS henceforth) and the labor demand- (LD henceforth) schedule in the (N,ω)-space. Show graphically the effects of a rise in τ on equilibrium hours worked, $N^{\star}$. Explain by using an economic reasoning.

Answer. The LD-schedule is downward-sloping since the MPL is decreasing as a result of diminishing returns to labor. To plot the LS-schedule, we use $\omega = \frac{\gamma}{(1-N^S)\,.(1-\tau)}$, which indicates that labor supply (horizontal axis) varies in the same direction as the before tax real wage (vertical axis). A rise in the tax rate τ shifts the LS-schedule to the left and thus reduces equilibrium employment. The reason is that a rise in the tax rate, τ, lowers the relative price of leisure which encourages agents to increase leisure and to work less (through the substitution effect).

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