Assignment Semester 1, 2023-24 Session

Q1 and Q2 - Week 1. From Pure Maths To The Maths Of Economics And Finance

Q1 Simplify $\exp\left(2\ln A + 3\ln B - 4\ln C + 5\ln D^6\right)$, where A, B, C, D are all positive quantities.

Q2 Revenue $R(P, Q) = PQ$ is a function of two variables: price P and quantity Q. Sketch a contour map of R in the region $0 \leq P \leq 10$, $0 \leq Q \leq 20$.

HINTS: (i) What mathematical shape is exemplified by the equation $xy = 12$? What would a contour map of the function $f(x, y) \equiv xy$ look like?

(ii) One suitable contour height for $R(P, Q)$ is $h = 50$. Pick other contour heights using your own judgment.

(iii) Note that the question asks for a sketch, not an exact plot.

Q3 and Q4 - Week 2: Things Economists Do With Differentiation

Q3 (a) By gathering berries for t hours, Robinson Crusoe can obtain $X(t)$ kg of berries, where $X(t) = 5\sqrt{t}$. How many kg of berries will he obtain in his fifth hour of labour?

(b) Robinson works continuously, starting at 8:00 am. At noon, what is his marginal product of labour-time?

Q4 (a) National income Y currently stands at \$20 tn/yr (trillion dollars per year) and consumption $C = C(Y)$ at \$16 tn/yr. If marginal consumption $C'(Y) = 0.7$, use the Small Increment Formula to obtain an approximation to the level of consumption if Y rises by \$0.2 tn/yr.

(b) The general price level P is rising over time according to the formula $P(t) = P_0(\exp(\alpha t) + bt)$, where P_0, α, and b are positive parameters. Calculate:

(i) The rate of increase of prices, $\dot{P}$ at t;

(ii) The rate of growth of prices $\hat{P}$ at t. Then

(iii) Evaluate both $\dot{P}$ and $\hat{P}$ when $t = 0$.

Q5 - Week 3: Unconstrained Optimization With A Single Choice Variable

Q5 Consider the optimization problem whose formal statement is

$$\min_x f(x), \text{ where } f(x) \equiv 3x^4 - 4x^3 - 12x^2 + 10 \quad [\text{P}]$$

(a) Write down the first-order condition for this problem.

(b) Find any critical points of the problem.

(c) By using the second derivative function $f''(x)$, decide which critical points, if any, are strict local minima of $f(x)$.

(d) Using the information obtained so far, solve the problem P.

Q6 and Q7 - Week 4: Many Variables. Constrained Optimization.

Q6 (a) Mr Jones has utility function $U(A, B) = AB$. He loses b bananas, reducing his banana holding to $B - b$. How many extra apples a will he need as compensation, to restore him to his former level of utility?

(b) From consuming A apples and B bananas, Ms Jones gets utility $U(A, B) = \sqrt{AB + A + 2B}$. What is Ms Jones' marginal utility with respect to apples, in terms of A and B ?

Q7 My utility function over apples (A) and bananas (B) is $U(A, B) = A^2B$. If apples cost \$6 per kg, and bananas cost \$4 per kg, and I have \$50 to spend. I wish to maximize utility, subject to the constraint that the value of my purchases equals by budget limit \$50.

(a) State my problem formally, as a problem of constrained optimization, and say what the objective function and choice variables are.

(b) Write down the corresponding Lagrangean function.

(c) Obtain first-order conditions, and state what other condition is needed to obtain a solution.

(d) Obtain optimal values A^*, B^*, explaining your working clearly. You may assume that $A^* \neq 0$ at the solution. You may also assume that the second-order conditions for this constrained problem are satisfied.

Q8 and Q9 - Week 5: Logic, Sets, and Functions.

Q8 (a) Express the set $\{x: x \in \mathbb{R}_{(+)} \wedge (x^2 < 16)\}$ using interval notation.

(b) Are the following three statements logically equivalent? Use the symbols and methods of propositional logic to find out, and explain.

i. Without a well-educated labour force, the economy cannot thrive.
ii. If there is a well-educated labour force, the economy can thrive.
iii. If the economy can thrive, the labour force must be well-educated.

HINT. Let e mean 'The workforce is well-educated'. Let t mean 'The economy can thrive'.

Q9. (a) What is meant by $[2,3)$? Explain informally, then formally, using bound-variable notation.

(b) What is meant by $\mathbb{R}_{(+)}$, and why is this set important in economic applications?

(c) From income Y dollars per year where $0 \leq Y \leq 10^6$, and leisure L hours per day, where $0 \leq L \leq 24$, I get utility $U(Y, L)$, which is a non-negative real number. State the domain and codomain of U, and use these to express U in colon/arrow notation. **HINT.** The domain and codomain are always <u>sets</u>.

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