**UNIVERSITY**

**DEPARTMENT OF MATHEMATICS**

# Comprehensive Research Questions

The questions that follow are designed to assess:

* CONTENT: Your understanding of important concepts and mathematical techniques taught in the required courses in the major.
* ANALYSIS: Your ability to see the connectivity (internal links) among the different areas of mathematics.
* RESEARCH: Your aptitude for extending your mathematical repertoire beyond what you have been taught – to deal with unfamiliar topics, but related to areas you have studied.
* COMMUNICATION: Your skill in communicating mathematics – to present your analysis in a clear and coherent manner reflecting the mathematical style and sophistication appropriate to your mathematical level.

**Directions:**

1. Answer five out of six questions.
2. Use at least three pages for each question; use only one side of the paper, double spaced. Word must be used, as well as MathType for math symbols. Use Times New Roman font size 12. Also, type (or copy and paste) in the question using bold font for each problem (the questions may remain single spaced). The parts of each question may be typed in (or copied and pasted), again using bold font, at the appropriate part in the paper.
3. Work independently. If you have any question, you may meet with any department faculty. Otherwise, you may not collaborate, give or receive any assistance of any form from anyone else. Failure to comply will result in a failing grade for this course, in addition to the other consequences for violating the Academic Integrity Policy.
4. Cite all the sources you use for the report. You may use the examples in the sources as a guide for any examples you give. Except for examples of proofs from Question 3, do not use the same examples in the sources. You must cite any examples you use or modify.
5. You will present one question to the faculty. As indicated in the syllabus, you may not present question #2. While your work should be the basis of your presentation, the presentation should not be a duplicate of your work.
6. When draft before presentation is approved, provide copies of each question to each faculty member. The copies should be provided at least one week before presentation. Also, reserve a room with the Registrar’s Office for the arranged time. Be sure you get a room equipped with the appropriate technology.
7. After presentation, make any final corrections suggested by faculty, and provide one more copy to the department. Also, this final copy is to be burned onto a CD or flash drive and submitted to the department. PowerPoint, or other presentation software, must be used for the presentation, this should be copied onto a flash drive and submitted to the department.

1) Differentiation and integration are considered inverse operations, but they were invented separately and have different geometric interpretations.

1. Give the formal definition of derivative. Using a specific example, compute the derivative using the definition. Show how the derivative can be interpreted algebraically and graphically.
2. Give the formal definition of integral. Using a specific example, compute the definite integral using the definition, or approximate the integral using a Riemann sum. Show how the integral can be interpreted algebraically and graphically.
3. Discuss at least two applications using derivatives. Use an example for each application.
4. Discuss at least two applications using integrals. Use an example for each application.
5. In probability theory, there are two kinds of random variables, discrete and continuous.
6. Explain the difference between discrete and continuous random variables. Give examples of each.

b) Answer the following questions on discrete random variables:

1. Describe the characteristics of a discrete random variable and its probability distribution.
2. How do you find the probability of an event involving a discrete random variable? Use an example to help illustrate.
3. What purpose do the mean, median, and standard deviation serve in describing the characteristics of the distribution of a discrete random variable? How do you find the mean, median, and standard deviation? Use the example in part ii) to help illustrate.
4. The binomial distribution is the most famous of the discrete distributions. Describe the properties of this distribution and how you find the mean, and standard deviation. How do you find the probability of an event? Provide an example.

 c) The study of the distribution of continuous random variables is the part of probability theory most closely related to calculus. Associated with each continuous random variable is a function called a probability density function. Answer the following questions on continuous random variables:

1. Define a probability density function. Give an example.

1. How is the probability density function used to find the probability of an event? Use example from part i) to illustrate.
2. Give formulas for finding the mean, median, and standard deviation for the distribution of a continuous random variable. Use example from part i) to illustrate.
3. Many important random continuous phenomena are modeled by a normal distribution. What is the probability density function for the normal distribution?
4. The art of mathematics is creating proofs. Just as a painter has some basic modes of painting, such as oils and watercolors; so the mathematician has some basic modes of proof.
* Proving  by direct conditional proof. We assume *P* with the explicit intention of deducing *Q*.
* Proving  by contrapositive. We assume  with the explicit intention of deducing , i.e., using the equivalence .
* Biconditional proof. Proving sentences of the type  using the equivalence

 .

* Proof by cases. Proving sentences of the type  using the equivalence

 .

* Proof by contradiction. A proof by contradiction of a sentence *P* is a proof that assumes  and yields a sentence of the type , i.e., using the equivalence

 .

a) Show that the pairs of logical statements in the last four bullets are logically equivalent.

1. Give two examples of each type of proof. You may select proofs from Calculus, Linear Algebra, Abstract Algebra, Geometry, Number Theory, Introduction to Real Analysis, and Foundations of Mathematics.There should be a variety of examples, i.e., the proofs should not be similar in form, and should include a variety of different topics. (Note: For these examples, you need not modify the proof, but be sure to cite the source.)
2. Direct conditional proof.
3. Conditional proof using contrapositive.
4. Biconditional proof.
5. Proof by cases.
6. Proof by contradiction.
7. The concept of infinity has been studied in Calculus, Modern Geometry, and Foundations of Mathematics. Discuss the uses of infinity in these courses as indicated below.
8. In Calculus, infinity has been used in some limits, improper integrals, sequences, and series. Give examples of the different ways that infinity is used.
9. In Geometry, points of infinity or ideal points were discussed in hyperbolic geometry.
10. In Foundations of Mathematics, infinity played a major role in sets and cardinality. Give examples of sets of each infinite cardinality.
11. A function is one of the key concepts in many areas of mathematics.
12. Define a function, a 1-1 function, an onto function, an inverse of a function. Provide examples for each as you define them.
13. Functions were also studied in various mathematics courses. In these courses, the functions had additional properties. Discuss these functions, including their additional properties, and provide examples. Note: Some of these functions may have different names in different courses.
14. Linear Algebra.
15. Abstract Algebra.
16. Modern Geometry.
17. Algebraic systems relate sets of elements with binary operations.
18. Describe the algebraic systems setting up a hierarchy beginning with groups, then abelian groups, rings, integral domains, and fields. Explain how each additional more restrictive system is an extension of a previous system with additional properties. Give examples for each of these five algebraic systems. Be sure to include examples of groups that are not abelian. Also, provide examples of rings that are not integral domains, and examples of integral domains that are not fields.
19. Consider the setwith operations matrix addition and multiplication. Discuss the properties of an algebraic system that has. Is a group and/or an abelian group? Is  a ring and/or an integral domain? Show, by examples, which properties of a field this algebraic system does not have.
20. Define a vector space. Is a vector space a group and/or an abelian group with respect to vector addition? Is a vector space a ring, integral domain, and/or field with respect to addition and scalar multiplication? Pay particular attention to the definitions of the arithmetic operations for each of the algebraic systems.